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ABSTRACT

This report summarizes a project to develop computational estimation materials for students in grades 6, 7, and 8. In chapter 1, the need for estimation skills is discussed and the goals of the project are stated: to develop a carefully sequenced set of lessons, activities, and maintenance work on estimation with whole numbers, fractions, decimals, and percent; to implement the program in grades 6, 7, and 8; and to evaluate the effects of the program on achievement in terms of skill in estimation and the types of processes employed. The second chapter describes the guidelines, issues, components, and pacing of the program. In chapters 3 and 4, findings from the evaluation of the pilot materials are presented, while chapter 5 describes the revision process. Seven appendices contain the pacing guide for the pilot materials; the attitude questionnaire and results; the mental computation tests and results; the computational estimation test, acceptable intervals, and results; the midyear test and results; a summary of the interview packet; forms used to evaluate the instructional materials; and a list of presentations made to disseminate project materials. (MNS)

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Developing Computational Estimation Materials for the Middle Grades

Final Report
National Science Foundation
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This project was made possible by a grant from the National Science Foundation. Developmental projects of this size and scope are needed to stimulate curricular change in mathematics programs throughout the country. It is our hope that the new instructional approaches to teaching computational estimation reflected in these project materials will provide a point of departure and help establish momentum for a national effort to rethink and revise mathematics programs.

We are indeed grateful to the National Science Foundation for their financial support of this curriculum development project. This research would not have been possible without help from many sources. We thank the following schools in Missouri and Illinois for their cooperation:

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South Junior High - Manchester, Missouri
Fanning Middle School - St. Louis, Missouri
Chute Junior High - Evanston, Illinois
Haven Junior High - Evanston, Illinois
Congress Park Elementary - LaGrange Park, Illinois
Cossett Elementary - LaGrange Park, Illinois
Park Junior High - LaGrange Park, Illinois
Wright Junior High - Lincolnshire, Illinois
Field Junior High - Northbrook, Illinois

In particular, we thank the teachers and students in those schools for participating in the project. Their efforts in using the materials and providing feedback were essential to the success of the project.

We thank Dr. Richard Lodholz, Mathematics Coordinator of the Parkway School District for serving as a local site director throughout the project. Special appreciation is due Patricia Conley, Michael Galligar, Michael Kocik and Robin Remich for their assistance in preparing the program materials. We also thank Ann Bestgen, Bronna Flanagan and Cathy Panetti for their efforts in scoring and analyzing the data throughout the various phases of the project.

Estimation skills must become an integral part of every forward looking mathematics program and this effort provides some viable ideas. The national interest shown in the instructional materials developed for this project together with the positive outcomes documented in this Final Report should make all of us feel that the hard labor was worthwhile. It is our sincere hope that these materials will help accelerate the development and use of improved instructional attention to computational estimation in our school mathematics programs.

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CHAPTER I -- INTRODUCTION

Proficiency with estimation has long been recognized as an important outcome for the study of mathematics. However, estimation has seldom received much emphasis in school programs and few curriculum materials exist on the topic. Computation, one important, widely-used type of estimation, is defined as "the interaction and/or combination of mental computation, number concepts, technical arithmetic skills including rounding and place value, and less straightforward processes such as mental compensation, that rapidly and consistently produce answers that are reasonably close to a correctly computed result. This process is done internally without the external use of a calculating or recording tool." The following example illustrates a practical application of computational estimation in daily life:

> You have only \$5.00 and want to purchase two cartons of milk at \$1.79 each and three loaves of bread at \$0.59 each. Do you have enough money?"

In the past ten years there has been renewed interest in the topic of estimation. Attempts to identify fundamental skills for all students and discussions of needed reforms in the mathematics curriculum have provided strong support for greater attention to estimation (Bell, 1974; NIE Conference on Basic Mathematical Skills and Learning, 1975; National Council of Supervisors of Mathematics, 1978). A major report of the

National Council of Teachers of Mathematics, *An Agenda for Action: Recommendations for School Mathematics of the 1980's* urged that "Teachers should incorporate estimation activities into all areas of the program on a regular and sustaining basis, in particular encouraging the use of estimating skills to pose and select alternatives and to assess what a reasonable answer may be."

The widespread use of estimation in many everyday situations involving mathematics is well recognized, including the fact that estimation is often more important and practical than exact computation for many everyday uses of mathematics. The widespread use of hand calculators gives added importance to the ability to estimate and recognize reasonable answers.

Despite its importance, estimation has been the most neglected skill in the mathematics curriculum (Carpenter, et al, 1976). A review of mathematics basal textbooks showed very little attention is given to the systematic development of computational estimation skills (Skvarcius, 1973). Another recent study of three widely used mathematics textbook series revealed that estimation appeared in less than three percent of the lessons (Driscoll, 1981).

The lack of attention to computational estimation has been documented by low performance of all age groups in three mathematics assessments conducted by the National Assessment of Educational Progress (Carpenter, et al, 1976, 1980, 1983). These poor performances were consistent with those reported earlier by the National Longitudinal Study of Mathematics Ability, (NSLMA) (Wilson, et al, 1968). In two summaries of research on estimation, Driscoll reported that little work has been done in the important area (Driscoll, 1981, 1982). He also pointed out that not only are estimation skills difficult to assess but such skills do not develop simply from taking more mathematics courses or through maturation.

A thoughtful examination of the evolution of computational estimation in school mathematics was provided by Buchanan in which he reported that "Instruction in estimation is not something that has been tried and failed; it has not been tried on any sustained or systematic basis." Although Buchanan's bleak assessment is accurate, some recent research efforts have provided new direction. In a study entitled "Identification and Characterization of Computation Estimation Processes Used by Inschool Pupils and Out-of-school Adults" some highly important stategies were detected from extensive interviews with good estimators (Reys, et al, 1982). This research suggested a general framework for instruction in estimation. This research, coupled with other recent research efforts on estimation cited by Driscoll is in the spirit voiced by one of the principal investigators who wrote that, "more must be learned in the next few years about how students develop these skills, how this work can be integrated into the curriculum, and how instruction can more closely fit the psychology of the learner..." (Trafton, 1978).

The growing pressure for more attention to estimation from professional groups, research findings on the types of strategies employed by good estimators, and the long-term interest in the topic by the principal investigators led to the development of this project entitled, "Development and Evaluation of Computational Etimation Materials in the

Middle Grades." It was funded by the Development in Science Education Division of the National Science Foundation and had the following three major purposes:

1. Develop a carefully sequenced set of lessons, activities and maintenance work on computational estimation with whole numbers, fractions, decimals and percent.
2. Implement the program in sixth, seventh and eighth grade classrooms.
3. Evaluate the effects of the program on student achievement in terms of skill in estimation and the type of processes employed.

The investigators felt that the existence of a comprehensive program in the form of classroom instructional materials would be useful to curriculum developers and researchers, and stimulate other long term efforts to improve the quality of computational estimation programs.

This report describes the development, implementation and evaluation of the pilot intructional materials. Also discussed is the revision of the materials. The revision used evaluation information to refine the program and package it in a form that could be readily used in other situations. The revised instructional materials for grades six, seven and eight are included as part of this final report.

CHAPTER II -- PROGRAM DEVELOPMENT

Planning and developing the program content and materials was a major undertaking. Since there had been little prior work on computational estimation, apart from a limited focus on computing with rounded numbers, much time was devoted to determining the content and approaches of the program. We also faced the challenge of developing a comprehensive, cohesive program within the limits of the time that schools could devote to estimation. The pilot program materials were developed in the spring, summer and fall of 1982. During the spring of 1982 it was possible to have a few classroom teachers informally test some of the approaches. Their comments and favorable feedback were helpful in guiding the development of the materials. Four areas of program development are now discussed.

Guidelines

Several guidelines were followed in developing the program. These included:

1. The program should be aimed at middle ability students. We believed it was critical for the materials to work well with the majority of students in order to attain the goal of making computational estimation a basic skill for most students.

2. The program should incorporate a variety of strategies and processes in order to enable students to deal quickly and efficiently with the many types of situations that occur when estimating. The findings from the Reys study, cited earlier, formed the foundation for the program.

Multiple strategies were to be built into the program, including front-end estimation, clustering, use of numbers that are easy to work with mentally and compatible numbers in division. The processes of adjusting estimates, compensation and recognizing sensible answers were to be emphasized throughout the program.

3. The program needed to address related number relationships and mental computation strategies that facilitate estimation.

4. Program materials needed to be in a format that would clearly present the content and promote student involvement through discussion. The materials further needed to communicate clearly to teachers the nature of the materials as well as the strategies with which teachers were not likely to be familiar.

5. The program needed to be limited to a reasonable amount of instructional time in order to be accepted by teachers.

6. Instruction in estimation is facilitated by placing estimation in real world contexts. This helps students see the usefulness of estimation and to become familiar with the many situations in which it is natural to estimate.

Issues

Several issues needed to be resolved during the development of the program including the amount of instructional time for the program, the problem of implementing a three-year program in one year and the format of the lessons. These issues are now discussed in more detail.

1. In our contacts with schools we indicated that the program would consist of 10 lessons throughout the year and take a total of about three

weeks, including testing time. It was essential that the program fit within the general framework of the amount of time that schools would be willing to devote to teaching computational estimation, so they would cooperate in using our materials.

However, it soon became apparent that 10 lessons would not be sufficient to cover the range of content to which estimation should be related, develop the program content carefully and thoughtfully, and deal with prerequisite skills and related skills and understandings. Thus, the decision was made to include several minilessons, which would take 5 to 10 minutes to teach. We also developed a series of maintenance worksheets to provide regular practice in estimating using the strategies taught.

2. It is well recognized that mathematical ideas and skills develop over time. We believed that confidence and competence in estimating could not be developed in a single year, especially in light of the limited contact that students likely had with estimation in previous years. Ideally, estimation instruction in grade six would establish a base for work in later grades. Thus the strategies, most of which would be new to students, would be reinforced and extended over a period of years.

Yet, the grant required that the materials for grades six, seven, and eight be implemented and evaluated in one year. We decided that repeating the same or similar lessons at each grade level would severely limit the scope of the program by omitting important topics or extensions of them.

The decision was made to remain true to the goal of developing a three-year program, and at the same time build in enough redevelopment of basic strategies to enable seventh- and eighth-grade students to be successful. This seemed to be a reasonable compromise, even though it remains a major limitation in the program.

3. Much consideration was given to the form the curriculum materials should take. Since many of the approaches would be new to teachers and students, we felt it was important that the material clearly communicate the main ideas to students and promote student involvement through discussion and practice work. We finally decided to produce the developmental portion of the lesson on overhead transparencies. The transparencies highlighted the key ideas and steps, listed questions, used real world settings, and had short exercise sets for students to do as part of the lesson development. This led to a develop-brief practice-reteach-follow up practice format for each lesson, which is consistent with the research for effective teaching as describd by Good and Grouws (1983).

Components

In order to provide materials that would help teach estimation strategies and processes effectively, several program components were developed. These components are discussed in this section.

Lessons : Ten full-period lessons were developed for each grade level. Each lesson focused on major strategies and processes of estimation, and included both teacher and student material.

Teacher Material Format:

1. Objective(s) - statement of lesson objective(s) in behavioral terms.
2. Teacher Background - discussion of content and

strategies to be taught, with additional comments designed to provide insight into the content and methodology of the lesson.

3. Teaching the Lesson - suggestions for teaching the lesson with guidelines for using the transparenies and key questions to ask.

4. Using the Exercises - brief comments on the use of the student assignment sheets.

5. Answer Keys - suggested range for estimation exercises.

Student Worksheets : Two- or three-page student worksheets accompanied each lesson. The worksheets provided practice with the approaches introduced in the lesson and often applied them in real world applications. Teachers were directed to use these materials for in-class practice and/or homework and encouraged to check and discuss them the following day.

Minilessons : Approximately 20 minilessons were developed at each grade level. Minilessons were designed to take 5 - 10 minutes to teach, with all work provided on one overhead transparency. Minilessons were designed to:

a. Develop prerequisite skills.
b. Teach additional strategies.
c. Present variations or extensions of strategies taught in lessons.
d. Develop mental arithmetic skills which are useful in estimation.

Maintenance Sheets : Maintenance sheets were developed for use after every two to four lessons. Their purpose was to provide cummulative practice with the strategies taught.

Pacing Guide

The purpose of the pacing guide, which specified the lessons, minilessons and maintenance sheets to be used each week, was to ensure the teaching of estimation on a systematic basis and to facilitate program monitoring by the investigators. Teachers had freedom to determine when to use the material each week and deviate from the schedule when necessary. One disadvantage of the guide was that estimation materials on a topic, such as whole number division, likely were not used when that topic was covered in the regular curriculum. A copy of the pacing guide for each grade level is shown in Appendix A.

CHAPTER III -- PROGRAM IMPLEMENTATION

The pilot estimation materials were implemented in 24 classrooms in Missouri and Illinois. Prior to discussing the comprehensive implementation efforts, special mention should be made about the pilot teachers. Implementing the program required a substantial commitment by the schools and teachers. In addition to devoting approximately three weeks of regular instructional time to the program, participation meant additional preparations by teachers with new content and their willingness to evaluate the program materials. The commitment and involvement of the teachers is a tribute to their professionalism and desire to improve the quality of school mathematics programs.

Design and Sample Selection

During the spring and summer of 1982 school districts and teachers were contacted about their willingness to participate. The project was carefully described, including the instruction and evaluation components of the program. We explained that the project was aimed at the "average" student. Thus in schools which utilized ability grouping, sections of advanced or low achieving students would not be selected. If schools used heterogeneous grouping, however, all students in the class were to participate in the program.

Five school districts agreed to participate as treatment centers. In these districts, 24 teachers agreed to pilot the materials. In each case treatment teachers were regular classroom teachers with four or more years of teaching experience. Another 24 teachers in the same or comparable districts were selected to be control classes. The distribution of the 48 classes is shown in Table 3.1.

Table 3.1

	Gr.6	Gr.7	Gr.8
Treatment	8	8	8
Control	8	8	8

In only one case, sixth grade, did a teacher have both a control and treatment class. This happened because this middle school split their students for mathematics and only one teacher taught all of the sixth grade mathematics. She reported no problems in maintaining the integrity of the treatment and control classes.

Seven school districts in Missouri and Illinois participated in the project. In five of the districts there were both control and treatment classes. The districts provided a broad spectrum of social-economic levels. Table 3.2 provides a brief profile of the participating schools and documents that a wide range of schools were represented.

Inservice Training

A 90-minute orientation meeting was conducted with all treatment teachers in late September in order to ensure a common level of understanding about the new approaches to estimation being implemented and

Table 3.2

Selected Characteristics of Participating Schools

District	School	Grades	Grades Used	Average Class Size	Percent Minority	School Type	Income Level
Columbia, MO	Fairview (T,C)	K-6	6	27	10%	Small City	High
	Jefferson (T,C)	7-9	7	29	15%	Small City	Medium
	Oakland (T,C)	7-9	8	26	5%	Small City	Medium
Parkway (MO)	Green Trails (T,C)	K-6	6	25	8%	Suburban	High
	South (T,C)	7-9	7	27	8%	Suburban	High
	East (T,C)	7-9	8	27	8%	Suburban	High
St. Louis, MO	Fanning (T,C)	6-8	6-8	34	40%	Inner City	Low
Evanston, IL	Chute (T)	6-8	6-8	32	35%	Small City	Medium
	Haven (C)	6-8	6-8	31	35%	Small City	Medium
LaGrange Park, IL	Congress Park (T)	K-6	6	23	10%	Suburban	Medium
	Cossett (C)	K-6	6	24	10%	Suburban	Medium
	Park (T,C)	7-8	7-8	25	10%	Suburban	Medium
Lincolnshire, IL	Wright (T)	6-8	6-8	22	3%	Suburban	High
Northbrook, Il	Field (C)	6-8	6-8	20	1%	Suburban	High

the importance of teaching estimation.

This workshop included:

1. Discussion of the nature and importance of computational estimation and key estimation strategies;
2. Overview of the testing program;
3. Demonstration of all program components: lessons, student worksheets, minilessons and maintenance worksheets;
4. Presentation of the pacing guide;
5. Distribution of estimation notebooks containing all instructional materials as well as additional resources in the form of key articles on estimation.

Ideally, it would have been desirable to have a longer time for initial training of the staff, but regular visits with pilot teachers were planned during the year. Furthermore, we felt it was important to determine if the materials would work well without extensive training of teachers, as the ultimate goal was the development of a program that could be used on a widescale basis.

Control teachers did not have inservice training other than a brief explanation of the project and the testing schedule. Control teachers were urged not to change their instructional emphasis on estimation. Thus, if they regularly taught estimation in their mathematics programs, they should continue to do so. Control teachers were not provided with project materials.

Monitoring of Treatment Classes

Each treatment teacher was visited briefly several times during the the year. During these visits lesson evaluation forms were collected and teachers were asked to share their impressions and reactions to the estimation materials based on their recent experiences. Problems in following the pacing guide were also discussed. This informal communication process provided valuable insights which supplemented the information provided in written form and led to a high level of teacher input throughout the project.

These visits not only reminded teachers of the project's dedication to the development of quality instructional materials but also encouraged teachers to stay on the proposed pacing guide. It was also felt that this regular monitoring of progress helped minimize disruption to the schedule when one of the participating school districts was on strike.

CHAPTER IV -- PROGRAM EVALUATION

The major thrust of this project was the development of new instructional materials for teaching estimation in grades six, seven and eight. A careful and thorough assessment of these materials was viewed as a very important consideration in this development and every effort was made to conduct an appropriate evaluation. Three dimensions were examined, including students experiencing the materials, teachers using the materials and selected national consultants reviewing the materials. This section identifies how these dimensions were evaluated along with a description and summary of specific results for each dimension.

Students

The primary source of student data for treatment and control classes were obtained in the fall (September 14-19, 1982) and spring (April 25-29, 1983). In addition a midyear test was given to the treatment classes during the week of January 17-24. In a year long project of this nature involving 48 classrooms, student attrition was anticipated. Some students left the district and others changed schools. There were also some students absent on days when information was collected. We made the decision to analyze only the performance of students for whom complete data existed. The first value in Table 4.1 reports the number of students tested in the fall in a class and the second value reports the number for whom complete data existed.

Table 4.1

Number of Students in Each Treatment and Control Class

	Grades		
	6th	7th	8th
	30 - 28	32 - 19	28 - 23
	16 - 16	32 - 20	28 - 26
	36 - 23	26 - 23	34 - 25
	24 - 21	27 - 21	23 - 17
Treatment Classes	27 - 18	28 - 22	28 - 22
	32 - 20	22 - 17	29 - 23
	26 - 18	36 - 23	27 - 13
	20 - 17	23 - 18	29 - 19
Total	211 - 161	226 - 163	226 - 168
	22 - 20	22 - 18	28 - 23
	28 - 26	35 - 20	26 - 22
	34 - 22	23 - 16	32 - 18
Control Classes	29 - 26	27 - 23	27 - 16
	18 - 10	25 - 17	23 - 17
	17 - 13	28 - 20	27 - 21
	25 - 16	26 - 16	21 - 14
	30 - 23	29 - 20	25 - 17
Total	203 - 156	215 - 150	209 - 148

Thus the first entry 30 - 28 under the 6th Grade Treatment Classes means that 30 students began the fall testing (in one treatment class) and complete end-of-year data were available on 28 of them. Table 4.1 shows that the attrition rate varied among both the treatment and control classes but overall it averaged about 25 percent.

The decision to analyze only data from students participating the entire year was based on the assumption that students leaving during the year were not statistically different than those who stayed. A comparison of pretest computational estimation scores between those students dropping out and those remaining in the program revealed no significant difference ($p > .01$) at any grade level. Consequently, all of the analysis reported reflects only students for whom complete data were available as reported in Table 4.1.

Three different instruments (Attitude Test, Mental Computation Test and Computational Estimation Test) were used for all students participating in this project. Students in the treatment classes also took a Mid-Year Computational Estimation Test. In addition, a few treatment students from each grade were individually interviewed to gain additional insight into their thought processes related to estimation. A copy of each instrument used along with directions for it's administration are included in Appendices B, C, D, E and F. Figure 4.1 highlights the various components of the student evaluation and provides the time schedule which was followed.

Figure 4.1

Time and Sequence of Project Testing

September 14-18	January 17-21	April 25-29
Pretests (Treatment and Control Groups)	Midyear (Treatment Group Only)	Posttests (Treatment and Control Groups)
Day 1		Day 1
1. Attitude	1. Computational Estimation	1. Attitude
2. Mental Computation		2. Mental Computation
Day 2		Day 2
3. Computational Estimation		3. Computational Estimation
4. Individual Interviews		4. Individual Interviews

The remainder of this student information section summarizes each instrument and the related findings.

Attitude This eight-item questionaire measured students' feelings about estimation (Appendix B). Since no quantative values were assigned to either the question or student responses, their responses were used only to provide a descriptive profile.

A complete summary of the results are also reported in Appendix B. The 73/79 in the first row for Statement 1 reports that 73 percent of the sixth grade treatment students said "Yes" to the statement "Estimation is something I think is very important." in the fall, whereas 79 percent of them said "Yes" to the same statement in the spring. Thus the treatment sixth graders showed a six percent increase during the year, while the control sixth graders showed a decrease of three percent, from 66 percent in the fall to 63 percent in the spring.

There were consistent changes in Statement 2, "Estimation is something I use outside of school." It is noteworthy that both treatment and control groups at each grade level perceived estimation of more use in the spring than fall. This may reflect the attention given to estimation from both instruction and testing. It may also reflect some maturation by the students which led them to realize the increased use of estimation in their daily lives.

Two statements "I like doing."(No. 4) and "I am good at doing."(No. 7) encouraged students to make a self apprasial of their estimation skill. The general decline from the fall to the spring for both treatment and control group students at each grade level was not only surprising but disappointing. These changes may reflect student reaction to the computational estimation tests on which all students at each grade level experienced difficulty. These changes may also reflect a general interest toward school and learning in the fall which often wanes during the latter part of the school year.

Additional examination of Appendix B reveals that consistent patterns are not always clear for each statement. Nevertheless the complete data reported in Appendix B provides many opportunities for other comparisons.

Mental Computation Mental computation is an integral part of estimation. Consequently the instructional materials prepared for this project often included mental computation activities. Since mental computation was being taught and practiced, it was felt that some evaluation of performance on mental computation should be included.

A timed test was prepared to measure student mental computation performance on all four basic operations. Two Mental Computation Tests (MCT) were constructed. The sixth grade MCT included only whole numbers. The seventh and eight grade MCT used the same test as the sixth grade, except it included additional questions involving fractions, decimals and percent (Appendix C). Field tests of the sixth grade MCT produced test-retest reliability estimates between .83 and .85. Similar field tests of the seventh and eighth grade MCT produced reliability estimates between .90 and .92. The same test used in the fall was used again in the spring for both treatment and control groups. In all cases students wrote answers to the open-ended questions in the consumable test booklet. Table 4.2 provides a summary of the pre- and post-test class means for the Mental Computation Test.

Table 4.2

Pre- and Posttest Class Means on Mental Computation Test

		Grade 6		Grade 7		Grade 8	
		Pre	Post	Pre	Post	Pre	Post
TREATMENT	CLASSES	15.39	16.68	20.72	29.83	28.27	41.45
		11.88	12.56	18.68	39.91	28.12	31.19
		9.91	14.00	25.56	32.69	21.04	35.40
		12.05	16.42	16.95	26.68	33.76	34.47
		9.06	9.76	14.19	26.24	23.05	24.00
		8.41	10.65	19.43	28.30	23.91	26.48
		9.67	10.67	20.45	26.30	27.15	37.69
		10.35	14.41	19.68	25.05	24.56	30.61
	Total	10.84	13.14	19.46	29.38	26.23	32.66
CONTROL	CLASSES	9.55	8.55	33.28	44.44	31.17	37.91
		16.58	18.08	16.80	23.95	30.23	37.18
		8.86	13.00	28.25	38.75	16.50	22.00
		15.08	19.46	27.75	39.10	25.38	32.19
		10.70	12.30	19.25	23.06	28.18	31.71
		10.15	11.92	23.61	23.39	28.76	36.67
		12.56	13.06	14.50	23.69	27.71	33.57
		7.41	7.00	33.50	32.80	40.88	39.00
	Total	11.36	12.92	24.62	31.15	28.60	33.78

An analysis of variance on the mental computation pretest scores was made to find if the two groups were significantly different at the beginning of the year. Although the pretest means of the control group classes were higher than the treatment group classes, the groups were not significantly different ($p > .01$).

An analysis of variance on the mental computation growth scores from the pre- to post-test was made and the results are reported in Table 4.3.

Table 4.3

Analysis of Variance of Mental Computation Growth Scores From Pre- to Post-test

	Source	SS	df	MS	F
6th	Treatment	2.21	1	2.21	.69 (p=.42)
	Error	44.70	14	3.19	
7th	Treatment	45.90	1	45.90	1.82 (p=.20)
	Error	352.42	14	25.17	
8th	Treatment	6.26	1	6.26	.31 (p=.31)
	Error	283.23	14	20.23	

These analyses show the treatment and control groups did not differ significantly on their mental computation growth scores.

Hand scoring of the MCT revealed that many students in the control groups estimated on the posttest. In general, this "transfer" would be viewed as favorable, however since the MCT required exact answers any estimated responses were scored as incorrect. For example, students were asked to record the result of 76 ÷ 29 (Item No. 3). A frequent but incorrect response was 110. This "transfer of estimation" phenomenon was not observed on the pretest and occurred almost exclusively in the control groups on the posttest. This negative transfer was disappointing, and indicated the need for clearer directions on the MCT.

Students should recognize when to estimate and when to compute exact answers. In fact, instruction should help students sharpen their judgment so their estimation skills are used wisely. Unfortunately the heavy attention given to estimation throughout the year may have established an "estimation mental set" for many students which was reflected in this test. This phenomenon penalized many students in the treatment group on the MCT posttest and produced lower growth scores on the mental computation test. This must be considered when interpreting the mental computation results, including the analysis reported in Table 4.3.

The above phenomenon makes the interpretation of an item analysis very risky. Despite this limitation, significant growth on some parts of the Mental Computation Test were observed. This growth was apparent on items which relied on mental computation but did not encourage estimation. For example, items such as these:

125 x 10 (Item No. 17)
40 x 60 (Item No. 19)
300 ÷ 5 (Item No. 24)
2400 ÷ 60 (Item No. 27)
1/2 of 60 (Item No. 41)
50% of 60 (Item No. 61)

showed significant improvement ($p < .01$) from the pre- to posttests for the control groups. A careful examination of Appendix C shows six items (Nos. 24, 27, 28, 37, 61 and 63) where at least a 25 percent gain from the pre- to posttest for every grade in which the item was given. These dramatic gains were observed only in treatment classes, whereas much smaller gains were found on the same items in the control classes.

Thus if only the class means are used in an analysis, no significant differences are reported. However a deeper examination reveals that many control group students "estimated" on the mental computation test. This lowered the treatment group scores and made differences more difficult to detect. A further examination of the item analysis for the mental computation test shows that growth did occur for both treatment and control groups. However dramatic shifts of 25 percent or more on an item were observed only in treatment groups.

Estimation The main thrust of this project was the development of instructional materials for teaching estimation. In order to assess the impact of this instructional effort an estimation test was constructed for each grade. Care was taken to create straight computation items (those containing only numerical data) and application items (those containing numerical data embedded in a real world context.) Table 4.4 provides a summary of the number for each grade.

Table 4.4

Number of Straight and Applied Estimation Items by Grade

	Grades		
	6th	7th	8th
Straight	20	30	30
Applied	20	20	20
Total	40	50	50

Many items (19 straight computation and 7 applied items) were appropriate and therefore used in all three grades.

The items included a balance of addition, subtraction, multiplication and division. They also reflected as much as possible the instructional attention given to various sets of numbers in a particular grade level. For example, whole numbers were included for all grades. Although some fraction items were included in sixth grade, fractions were emphasized more in seventh and eighth grade. Percents appeared only in the seventh and eighth grade. A copy of each item and the grades in which it was used is reported in Appendix D.

The assessment of computational estimation skills is very challenging. Although different ways to assess estimation exist, each of them has some weaknesses (Reys and Bestgen, 1981). Despite some limitations, the assessment approach used in this project reflects techniques successfully used in earlier research (Reys et al, 1980). Each of the test items was produced on a 35mm slide and the items were shown sequentially using a carousel slide projector. This organization allowed for group administration and controlled the pace by allowing only a fixed amount of response time (12 to 14 seconds) for each item. In retrospect, our observations (which were collected with the interviews) suggests that students in the treatment group would have benefited greatly from an increase in response time.

All of the items were open-ended. Students were directed to write their estimates on a specially prepared answer sheet. A 6 cm x 35 cm answer sheet provided adequate space for the open-ended answers for the straight computation items on the front and application items on the back. It was purposely designed to be very compact to avoid any open space for students to either record the problem or do paper/pencil computation.

The open-ended format necessitated the construction of acceptable response intervals. These intervals reflected the strategies being taught. The acceptable interval for each item is also reported in Appendix D.

Pilot testing of the items was done to check on visibility by students throughout the room and clarity of context. The test-retest reliability estimates for the tests ranged from .78 to .88 in grades six, seven and eight. Since testing was done at different sites, it was important that directions and procedures be as uniform as possible. All texts were administered by the principal investigators and in every case a uniform set of testing procedures was followed.

In order to compare the estimation performance of the matched treatment and control classes, several analyses were done. Class means on the Estimation Pretest are reported in Table 4.5.

Table 4.5

Treatment and Control Pretest Means on the Estimation Test

	6th		7th		8th	
	T	C	T	C	T	C
	9.04	3.50	8.78	17.61	10.64	15.09
	4.88	13.38	4.55	4.30	11.81	12.18
	2.35	4.14	10.69	15.94	6.24	4.28
	8.69	11.00	11.36	14.25	18.65	17.88
	4.18	4.00	8.90	10.50	12.91	12.53
	3.06	2.77	9.74	10.70	11.74	13.90
	3.76	6.44	10.80	6.25	12.15	9.79
	6.06	2.68	7.53	12.20	9.00	13.50
Composite Class Mean	5.25	5.99	9.04	11.47	11.64	12.39

It shows the corresponding control means for the classes selected to match the treatment classes from the same school. The bottom line entry of composite class means shows that at each grade level the grand mean for the control group exceeded treatment group.

An analysis of variance was done on the pretest estimation means to determine if the groups were equivalent when the project started. The results for each grade level are reported in Table 4.6.

Table 4.6

Analysis of Variance on Estimation Pretest

	Source	SS	df	MS	F	
6th	Treatment	2.10	1	2.10	.18	(p=.67)
	Error	159.08	14	11.36		
7th	Treatment	23.52	1	23.52	1.83	(p=.20)
	Error	180.35	14	12.88		
8th	Treatment	2.26	1	2.26	.16	(p=.70)
	Error	201.72	14	14.41		

Since these results were not significant, the groups were assumed equivalent and covariance procedures were not used in the subsequent analysis.

We felt that both treatment and control groups would improve their estimation performance during the school year. The control groups experienced whatever attention the teachers chose to give estimation. Furthermore since every textbook series purports to teach estimation, it was hypothesized that some improvement in estimation would be demonstrated by the control classes. Since the students in the treatment classes received instruction in the project materials, improvement in their estimation skills was also anticipated. Therefore the main question addressed in this phase of the evaluation was whether the growth of the treatment and control classes was equivalent.

In order to address this question, an analysis of variance on the mean growth of the treatment and control classes was made. The pretest estimation scores for a class were compared with their respective posttest estimation scores to determine the growth. The results of the analysis are summarized in Table 4.7.

Table 4.7

Analysis of Variance of Growth in Estimation

	Source	SS	df	MS	F
6th	Treatment	39.75	1	39.75	4.27 (p=0.06)
	Error	130.38	14	9.31	
7th	Treatment	81.14	1	81.14	15.25 (p=.002)
	Error	74.51	14	5.32	
8th	Treatment	97.66	1	97.66	12.57 (p=.003)
	Error	108.77	14	7.77	

At each grade level, the growth of the treatment classes was greater than the control classes but as Table 4.7 shows, the level of significance varies.

Figure 4.2 provides a visual illustration of the growth differences between the treatment and control groups. It shows that individual class mean growths were much higher in the treatment classes than the control groups. The consistency of these findings across grade levels confirms the positive effect of the project materials in developing estimation skills.

An item analysis was done on both the pre- and post Estimation Test. It is also included in Appendix D and further substantiates that although

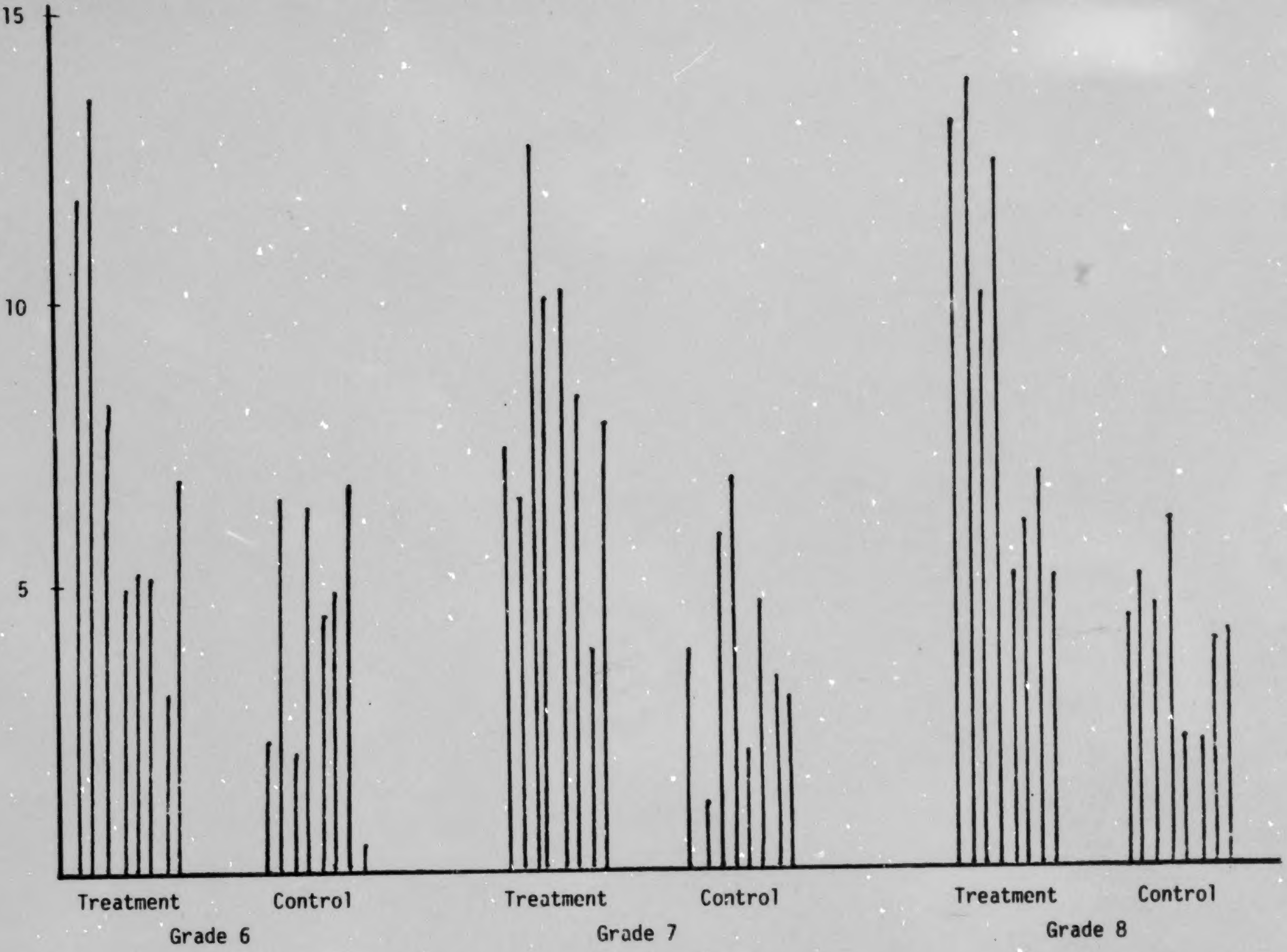

Figure 4.2

Growth from Pre- to Post Estimation Test estimation performance in both treatment and control group improved, students in the treatment groups improved more. An examination of the individual items shows that improvement for treatment students was observed on every item at every grade. The same could not be said for the control group, where slight drops in performance on some items occured (i.e., 6-7-A, 7-3-A, 8-3-A).

Perhaps the most interesting finding relates to dramatic shifts of performance (25 percent improvement or more) observed from the pre- to posttest. An examination of Appendix D shows improvements of 25 percent or more on each of the following items:

6th	7th	8th
6- 1-A	7- 4-A	8- 3-C
6- 5-A	7- 6-A	8- 6-C
6- 6-A	7- 7-A	8- 7-C
6- 9-A	7- 3-C	8-10-C
6- 4-C	7-10-C	8-11-C
6- 6-C	7-15-C	8-14-C
6- 7-C	7-16-C	8-15-C
6- 8-C	7-22-C	8-16-C
6- 9-C		8-17-C
6-10-C		8-21-C
6-11-C		8-22-C
6-15-C		8-29-C
6-20-C		

Although the number of items varied by grade, it is remarkable that such dramatic shifts were made on so many items (33 percent for grade 6, 16 percent for grade 7, 24 percent for grade 8). This finding is even more impressive when it is noted that there was not a single item at any grade in the control groups which showed a 25 percent improvement.

Appendix D offers opportunity for careful study and interesting observations. For example, although tremendous growth by students in the treatment group has been documented, much room for additional improvement remains. Far too many items remained very difficult for the students in the treatment group.

The pre- and posttest measures provided all of the comparative data between treatment and control groups for this project. In addition, a midyear paper-pencil estimation test was given to all treatment classes. The purpose of this midyear test was to provide an interium report on student progress. It also was designed to encourage a continuous high level of teacher involvement and adherence to the project schedule. Criterion referenced tests were constructed to reflect the specific strategies being taught in each grade level (Appendix E). In addition, the test included some questions to check on the recognition of sensible answers. It also provided opportunities to adjust estimates and compare them with exact answers. Each page of the test was carefully timed and required about 20 minutes to administer. An examination of the item analysis reported in Appendix E reveals that student performance was consistently above 60 percent on each exercise. We viewed this as a very strong performance level and an indication that the instructional program was being implemented. Results from these tests were returned to teachers and students so they would have some feedback on their progress in developing estimation skills. Interpretation of the scores was left to individual teachers.

Interviews In adddition to the group measures, individual interviews were conducted with some students at the beginning and ending of the school year. Test scores provide limited information about the cognitive processes students use or how these processes may have changed as a result of instruction. The interviews were designed to provide insight into the estimation processes and techniques actually used by students in problem situations. The particular students interviewed resulted from a random sampling plan, subject to the following conditions:

1. 12 students were interviewed from each grade.
   (Six at each site)
2. At least one student was interviewed from each treatment class.
3. Students from the top, middle and bottom one-fourth on the CET were interviewed.

An interview containing 10 problems was prepared for each grade (Appendix F). Several of the problems were from the CET. They were all open-ended and were selected to represent different estimation strategies. Five problems were common to each grade level. Throughout the interviews, which lasted between 15-25 minutes, students were encouraged to describe in their own words the procedures being used in making estimates. Each interview was audio taped and a transcript of their responses was made.

The purpose of the interviews was to help document some important process changes that were often observed and which could be attributed to the instruction in estimation. Although treatment teachers shared many examples of how their students thinkinq about estimation was altered, these interviews provided additional evidence of several interesting patterns. Three of them are now discussed:

1. Clearer understanding of the process of estimation.

Many students misunderstand estimation. For example, when asked to estimate, students will compute an exact answer and then use it to make an estimate. This happened frequently in the initial interviews. Here is how Holly, a sixth grade girl, responded to the problem:

$$\begin{array}{r} 729 \\ -371 \\ \hline \end{array}$$

She said "that's 358, so my estimate is 400." Holly revealed the same approach on another problem (2548 43) when she said "I can't find the answer to make an estimate." In each case, Holly felt that she needed an exact answer before she could give her "estimate."

Holly showed remarkable improvement in her estimation skills during the year. For example, her end of the year response to the problem (2548÷ 43) was "I changed it to 2400 divided by 40 and got 60. Or I could also change it to 2800 divided by 40 and get 70." Holly no longer felt the need to get an exact answer and this helped her realize that estimation is indeed a powerful process. Holly voiced this feeling about estimation when she said "I learned new things to use when estimating. It's faster and easier than it used to be." This same point was made by an eighth grader who at the close of the year said "Estimation makes things a lot quicker. It is easier than using paper because you don't have to write anything down. All you have to do is get close." These comments provide a natural lead to another important change which was observed.

2. Developed a greater tolerance for error.

As we have just noted, the strong desire to get exact answers can be a stumbling block for producing an estimate. One of the biggest challenges in estimation is for students to provide (and be satisfied with) an answer which they know is not exact. Years of experience in arithmetic has conditioned them to get "the right answer." This lack of tolerance for error appeared in several different problems. for example:

No. 1 "My estimate is 160003." (6th grader)

No. 5 "About 8988 because I just did it approximately." (7th grader)

Notice that in each case the units digit is the same as it would have been with traditional written algorithms.

The same students at the end of the year responded:

No. 1 "About 170000" (6th grader)

No. 5 "It's more than 7200." (7th grader)

Such comments suggest that these students changed during the year and felt more comfrontable with their estimates. This willingness to tolerate error in estimates is reflected in these comments:

> "Estimation is like getting close to the exact answer." (6th grader)
>
> "A lot of times when you estimate, you just leave out or change some numbers." (8th grader)

The process of adjusting (adding something on or taking something off) to produce a better estimate goes hand in hand with a tolerance for error. Developing this tolerance takes time. Although one year is not enough, these comments show that progress can be made.

3. Improved understanding of number concepts.

Fractions and decimals create confusion for many students. Lack of understanding of these number concepts often leads to confusion and errors with traditional algorithms. For example, consider these students response when asked to estimate the sum of 12/13 + 7/8 (No. 2):

> "I don't know what to do." (6th grader)
>
> "19/21, but that's an exact answer. My estimate is 20/20 or 20." (7th grader)
>
> "That's 20 over 20 or 1." (7th grader)

These responses reflect not only uncertainity about what to do, but a lack of basic conceptual understanding of fractions. The most frequent response by all students interviewed on this problem was to add numerators and denominators and report 19/21 as their estimate. No students verbalized anything about the relative sizes of the fractions, which is fundamental to estimating.

By the end of the year, many students took a global look at the numbers before applying an algorithm. This is evidenced by these comments:

> "They are both about one...so my estimate is 2." (6th grader)
>
> "12/13 is almost one and 7/8 is about one, so my estimate is almost 2." (6th grader)

> "They are both a little less than one so my estimate is a little less than two." (7th grader)

Realization of these fractions being close to one suggests that students' concept of fractions had improved. Furthermore their ability to take the global view of this problem represents a vital step in the estimation process.

It is impossible to capture and describe every important change suggested by these interviews. However several other patterns were observed. Students responded quicker, and often mentioned that estimation was easier because you didn't have to use "messy numbers." Many of the strategies taught during the year were not only used in the end of year interview, but were identified by students as they were being used. This allowed many students to call upon more than one strategy on a single problem. Rarely was more than one approach used at the beginning of the year.

These interviews confirmed that progress was being made in changing how students think about estimation as well as how they estimate. These interviews revealed not only a wide range of levels of development of estimation skills and strategies, but made it clear that additional instructional effort is needed. Clearly any instructional effort addressing estimation must be viewed as a multiyear effort.

In order to further capture the spirit of these interviews and provide documentation to interested parties, several interviews were video taped. The same students were interviewed in the fall before instruction began and then again in the spring after the lessons were completed. A composite videotape illustrating some of these process changes was made. It illustrates how students approached and solved some of the estimation problems in the fall and later in the spring after instruction was complete. This 3/4 inch cassette videotape is entitled "Improving Estimation Skills Through Instruction" and is available from the National Science Foundation and Robert E. Reys, 212 Education Building, University of Missouri, Columbia Missouri 65211.

Teacher

In addition to evaluating the effect of the materials through monitoring student performance, feedback on all of the instructional materials was gathered from the treatment teachers. Special evaluation forms were prepared for the materials (Appendix G). Teachers were asked to complete their evaluation of the instructional materials immediately following their use. The evaluation forms were designed to provide helpful information without burdening teachers with tedious paper work. This written feedback was supplemented with informal conversations held with treatment teachers throughout the year.

Teachers using the materials were in an ideal situation to recognize troublesome areas and problems that should be addressed in revisions. The teachers were generally conscientious in their review of instructional materials. They indicated places where directions were unclear, areas where the lessons were too long, and content that needed further development. They also gave specific examples of student experience and often offered suggestions for improvement and revision of materials. All of these comments greatly aided the revision process.

Feedback was by and large both encouraging and enthusiastic. This project not only helped teachers realize the importance of teaching estimation but alerted them to its natural integration with other mathematics topics. For example, several teachers commented on how the instruction on estimation reinforced and complemented the material in the regular curriculum. This comment surfaced repeatedly in the fraction work at all grade levels. One teacher commented, "My students seem to understand fractions better and that is helping them compute with fractions. They also are more sensitive to unreasonable answers when they compute."

Another positive feature of this estimation curriculum cited by teachers was that it allowed for and encouraged discussion within the math class as the following comment indicates " . . . the examples on the transparencies sparked a lot of discussion of whether answers should be exact or estimates and if estimates, what reasonable ranges should be considered." Teachers also mentioned that although initially students were uncomfortable giving answers which were not exact, as the instructional sequence progressed they became more comfortable with the importance of estimation as a real life, necessary and efficient process.

One of the principal investigators made arrangements with an eighth grade teacher to teach the ten project estimation lessons during the year in order to gain additional insights about the strengths and weaknesses of the program. The class was from a school not providing either treatment or control classes, and the results from this class were not used in any data analysis. The principal investigator taught all of the lessons and the regular teacher handled the follow-up homework discussion as well as the minilessons and maintenance sheets. The timetable paralleled the regular lesson schedule followed by all the treatment classes. This arrangement worked very well and provided valuable insight for the lesson revisions.

National Consultants

Information to improve and refine the instructional materials was sought from one other source. Several national consultants (Mary Lindquist, National College of Education; Joseph Payne, University of Michigan; and James Wilson, University of Georgia), themselves experienced teachers, examined the instructional materials and offered suggestions. Each of them reviewed the set of materials independently. They were asked to not only examine individual parts of the program but to consider the big picture of organizing and sequencing the materials. This allowed them to provide overall views of the materials and supplement the more microscopic examinations provided by teachers using the materials in classes.

Many comments from the consultants were very supportative of our effort. They were generally impressed with both the quality and quantity of the instructional materials which had been developed. One said "I would be delighted if all sixth, seventh and eighth grade students in the U.S. could be taught these lessons."

Although such positive comments from the consultants were encouraging, their feedback also contained many important recommendations and valuable suggestions for improvement. Some addressed broad issues, such as the organization and composition of the lessons. Some concerns were expressed, such as ". . . you press too soon for an estimate that is too precise . . ." and recommendations, such as " . . . applications should be emphasized more" were offered. In addition,specific ways to revise and improve individual lessons were identified. The consultants were both thorough and comprehensive in their reviews. Their responses were carefully studied and seriously considered in rewriting the final project materials.

The information from treatment teachers and national consultants provided the data base for program revision. All of these data were synthesized and used by the investigators as the instructional materials were revised.

Summary

Overall, the results of the classroom implementation of the pilot program indicate that the first effort at developing a comprehensive program for computational estimation built on multiple strategies resulted in significant growth in the estimation skill and thinking of students and was accepted by teachers. Of equal importance it provided essential information necessary to revise and refine the program.

Four factors should be kept in mind in considering the findings:

1. The results measured the effectiveness of a three-year estimation program after one year of implementation. Major shifts in the thinking and performance of students occur after repeated exposure to a topic for a period of years. We noted initially that students were hesitant about estimating and quite rigid in their thinking. It was clear that their limited contact with estimation in prior years had led them to be

mechanical in how they estimated, unclear about the purpose of estimating and to view estimation as a chore. It took several lessons for students to become more flexible in their thinking and more comfortable with estimation.

The fact that estimation performance grows over time was noted by a sixth-grade pilot teacher who taught a seventh grade class composed of students who were in the estimation program last year and those who were not. She reported observing a major difference in the estimation proficiency of those who were part of the pilot program and those who were not.

2. The program was implemented in 24 classrooms in public schools using the regular classroom teachers who had not received extensive training in teaching estimation. It was designed for use in a wide variety of classrooms under normal teaching conditions. Although we felt strongly that the program should be taught and evaluated in such a manner, obviously some control is reduced under such conditions. While the pilot teachers were conscientious and faithful in their teaching of the program, the level of commitment and instructional attention to estimation did vary. Time was often a factor, due to demands of the regular curriculum and interruptions in the normal schedule for special school events. In a few cases it was not possible for teachers to cover all minilessons and maintenance worksheets.

3. Problems remain in group testing of estimation performance. The use of slides under timed conditions for each item, as was done with the Computational Estimation Test, is a major improvement over conventional

testing. In many cases, however, the time allowed (12-14 seconds) was not enough. This response time was not enough to produce an acceptable estimate, to analyze the situation, select an appropriate strategy and produce an acceptable estimate. Each of the investigators noticed a major difference in how the treatment students approached the April administration of the Computational Estimation Test, as compared to their initial exposure to the test in September and the post-testing of the control students. On the posttest, treatment students approached the items with far greater confidence, indicating the feeling that they could be successful. This is in marked contrast to their first experience when one could see students giving up quickly or showing dismay when difficult items appeared.

That the CET may have undermeasured performance of students is supported by the student interviews. In the final interviews, most students were generally successful on the items, but often needed longer than 15 seconds to consider the situation, reflect and produce an estimate.

While the 12-14 seconds is necessary for many items for which the mental use of paper-and-pencil algorithms is feasible, additional time needs to be allotted for other items. Increasing time allotments, however, increases the liklihood that at least some students will use regular computational approaches.

4. Strong support for program effectiveness comes from informal observations of teachers and interviews and observations of students. As noted earlier, the final interviews provide evidence of gain in estimation

thinking and skill. Also, the way students approached the April administration of the CET, described above, showed a change in their perception of their ability to estimate. This is also supported by the videotape, "Improving Estimation Skills Through Instruction" cited earlier.

Several teachers noted a transfer effect to regular classwork, where students were more sensitive to answers that were not reasonable. While such evidence is not sufficient to demonstrate program effectiveness, it supports, and perhaps extends, the quantitative findings of the evaluation.

CHAPTER V -- PROGRAM REVISION

Originally we did not foresee that the revision of the pilot materials would be a major task. Several factors, however, led us to undertake a major revision of the initial materials in order to refine them, increase their effectiveness, and produce a systematic set of instructional materials that could be effectively used by teachers across the United States. These factors include:

1. The need to address more explicitly some factors related to the process of estimating, such as decision making and the development of an estimation "mind set."
2. Knowledge gained about teaching strategies and processes in developing and field testing the pilot materials.
3. Program considerations related to effective dissemination and widespread implementation of the instructional materials.
4. Feedback from teachers and consultants, our own observations on the use of the materials and achievement information.

Although the basic approaches to estimation and format of the materials remain unchanged, several major modifications of the pilot materials were made in the revised materials. The major changes are discussed here.

1. An increase in the number of lessons from 10 to 15 per grade.

As discussed earlier in this report, we felt that it was necessary to hold to the original plan of 10 lessons per grade in the pilot edition. This meant that some major strategies needed to be developed rapidly, and other strategies and extensions of strategies had to be handled as minilessons. The five additional lessons permitted major strategies to be developed more thoroughly and made it possible to consolidate the estimation minilessons into the 15 lessons.

2. Elimination of work on mental computation and reduction of the emphasis on prerequisite skills.

This was a difficult decision since both of these areas support and facilitate estimation. There did not seem, however, to be any way to give the areas the attention they require and have the necessary time to develop the estimation work carefully and thoughtfully. Mental computation needs to be addressed in the curriculum, but should be the focus of a project devoted explicitly to it. Some important prerequisite skills are incorporated in lessons and others are discussed in the teacher notes.

3. Elimination of minilessons and maintenance sheets.

The number of different components in the pilot program, specifically the minilessons and maintenance sheets, tended to make the program difficult to manage in the classroom. The five additional lessons and the decisions about mental computation and prequisite skills made it possible to eliminate the minilessons. It might have been desirable to retain the maintenance sheets, which provide cummulative practice, but the 15 lessons and 30 worksheets already provide a comprehensive program consuming at least three weeks of instructional time.

4. Less emphasis on precise estimates.

Several estimation strategies, such as front-end addition, subtraction and multiplication provide estimates that are quite close to exact answers. In the pilot edition, we were concerned about placing too strong an emphasis on precise estimates. Such an approach can make estimation seem more complex and difficult to students than it should. In many situations, such as checking of paper-and-pencil computation, calculator work and daily uses of estimation, "ballpark" estimates are sufficient. Also, the mental steps necessary in processing numbers to obtain precise estimates may exceed the capabilities of average and below-average students. This concern of pushing for too much precision in estimation too quickly was also shared by our consultants' review of the materials. Thus, in revising the materials, we placed less emphasis on precision throughout the program. We also tried to make a better distinction between estimation goals for all students and those for more capable students.

5. Greater attention to processes associated with estimation.

In addition to proficiency in using strategies to obtain estimates, a good estimator must develop an appropriate mental set and understand variables associated with estimating. These include such factors as:

a. Recognizing when an estimate is all that is required;
b. Distinguishing between when a "ballpark" estimate is sufficient and when it is important to get closer;

c. Selecting an appropriate estimate for a situation (mental flexibility);

d. Recognizing whether an estimate is an overestimate or an underestimate, when appropriate;

e. Adjusting a "ballpark" estimate to get closer;

f. Using estimation to determine when an exact computation is sensible or reasonable.

While these factors were treated in the piloted materials, our reaction and that of the consultants was that they needed more explicit attention. Thus the revised materials place greater emphasis on them. In Grades 6 and 7, the first transparency of each lesson, entitled "Get Your Mind in Gear", was devoted to these estimation processes. In Grade 8, where the problem of content coverage was greater, the processes were dealt with in the context of each lesson.

The program revision was time consuming. Our desire, however, to produce a program that would work well in classrooms and serve as a guide for future researchers and curriculum developers led us to produce a major revision of the materials.

In closing, we feel this project provided a much needed structured approach to teaching estimation. The project has not only developed some creative instructional materials, but has provided research data which documented its effectiveness in imporving students computational estimation performance. It has illustrated some new directions for teaching estimation which we hope will be seriously considered by those who help shape future mathematics curricula. Much work remains to further refine and extend these ideas both downward into earlier elementary grades and upward through the secondary school. Hopefully our work will stimulate others to offer further improvements toward teaching the important and challenging topic of computational estimation.

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APPENDIX A

Pacing Guide for Grades 6, 7, and 8 Pilot Materials

PACING GUIDE

GRADE 6

LESSON	TOPIC/STRATEGY		MINI LESSON	TOPIC/STRATEGY	
1	INTRODUCTION				
2	FRONT-END 3,4-Digit Numbers	(+)			
3	FRONT-END, AVERAGING Larger Numbers	(+)			
			1	Compatible Numbers Sums close to 100	(+)
			2	Compatible Numbers "Nice" dollar amounts	(+)
4	FRONT-END 3,4-Digit, Larger Numbers	(-)			
			3	Mental Computation Multiples of 10, 100	(+,x)
5	FRONT-END 1 x 3D, 1 x 4D	(x)			
6	ROUNDING 2^{+}D x 2^{+}D	(x)			
			4	Compatible Numbers Round one factor to 100, 1000	(x)
			5	Mental Computation 1 x 2D, 1 x 3D	(x)
7	SIZE OF QUOTIENT, FIRST DIGIT IN QUOTIENT 1D Divisors	(÷)			
8	COMPATIBLE NUMBERS 1D Divisors	(÷)			
9	SIZE OF QUOTIENT, ROUNDING COMPATIBLE NUMBERS 2D Divisors	(÷)			
			6	Recognize, Create Compatible Numbers 2D Divisors	(÷)
10	COMPATIBLE NUMBERS 2D Divisors	(÷)			

LESSON	TOPIC/STRATEGY	MINI LESSON	TOPIC/STRATEGY	
		7	$\frac{a \times b}{c}$	(x,÷)
		8	Mental Computation N ± Multiple of 10, 100	(+,-)
		9	Mental Computation 2D Numbers	(+)
		10	Mental Computation 2D Numbers	(-)
Whole Numbers				
		11	Identify Fractions Close to ½ and 1	
		12	Rounding Proper Fractions	(+)
		13	Front-End Mixed Numbers	(+)
		14	Find Fractional Part 1/2, 1/3, 1/4 of N	(÷)
		15	Compatible Numbers	(÷)
Fractions				
		16	Front-End, Rounding Decimals: Tenths, Hundredths	(+,-)
		17	Identify Decimals Close to 1 and 10	
		18	Compatible Numbers Round One Factor to 1 or 10	(x)
		19	Substitute Fraction for Decimal $0.25,\ 0.33\frac{1}{3},\ 0.5$	(÷)
		20	Compatible Numbers Substitute Fraction $0.25,\ 0.33\frac{1}{3},\ 0.5$	(÷)
Decimals				

PACING GUIDE

GRADE 7

LESSON	TOPIC/STRATEGY		MINI LESSON	TOPIC/STRATEGY	
1	INTRODUCTION, FRONT-END 3, 4-Digit Numbers	(+,-)			
			1	Averaging	(+)
			2	Front-End Larger Numbers	(-)
			3	Compatible Numbers	(+)
			4	Mental Computation Multiples of 10, 100 and 1000	(x)
2	FRONT-END 1 x 3D, 1 x 4D ROUNDING 2^+D x 2^+D	(x)			
			5	Mental Computation 1 x 2D, 1 x 3D	(x)
			6	Mental Computation Multiply by 50, 25	(x)
			7	Compatible Numbers Round to "Nice" Numbers	(x)
			8	Relationship between digits in factors and digits in product	(x)
3	SIZE OF QUOTIENT FIRST DIGIT IN QUOTIENT 1D, 2D Divisors	(÷)			
			9	Recognize, Create Compatible Numbers	(÷)
4	COMPATIBLE NUMBERS 2^+D divisors	(÷)			
			10	Compatible Numbers $\frac{a \times b}{c}$	(x,÷)

Whole Numbers

LESSON	TOPIC/STRATEGY		MINI LESSON	TOPIC/STRATEGY	
			11	Identify Fractions Close to 0, ½, or 1	
5	FRONT-End Fractions, Mixed Numbers	(+,-)			

LESSON	TOPIC/STRATEGY		MINI LESSON	TOPIC/STRATEGY	
			12	Mental Computation Mixed ± Whole Number	(+,-)
			13	Rounding Mixed Numbers	(+,-)
			14	Compatible Numbers Mixed Numbers	(+,-)
6	COMPATIBLE NUMBERS Unit Fractions: 1/4 of N	(÷)			
			15	Mental Computation Whole x Mixed	(x)
7	ROUNDING, COMPATIBLE NUMBERS Mixed Numbers	(x,÷)			
Fractions					
8	FRONT-END, ROUNDING COMPATIBLE NUMBERS Decimals	(+,-)			
			16	Mental Computation Multiply by 10, 100, or 1000	(x)
			17	Mental Computation Divide by 10, 100	(÷)
9	FRONT-END, ROUNDING COMPATIBLE NUMBERS Decimals	(x,÷)			
			18	Identify Quotients Greater, Less Than 1	(÷)
Decimals					
			19	Substitute Fraction for Percent 1/2,1/3,1/4,1/5,1/10	
			20	Mental Computation 1% or 10% of a Number	(x)
10	COMPATIBLE NUMBERS 1%, 10%, 25%, 33 1/3%, 50%	(x,÷)			
Percent					

PACING GUIDE

GRADE 8

LESSON	TOPIC/STRATEGY		MINI LESSON	TOPIC/STRATEGY	
1	INTRODUCTION FRONT-END (Whole Numbers, Decimals)	(+)			
			1	Compatible Numbers Sums Close to 100	(+)
			2	Mental Computation Multiples of 10, 100	(+)
			3	Mental Computation 2D, 3D Numbers: N + 19	(+)
2	AVERAGING (WN, FR, DC)	(+)			
			4	Mental Computation Chain Computation Multiples of 5, 10, 100	(+,-)
			5	Compatible Numbers "Nice" dollar amounts	(+)
			6	Mental Computation Multiples of 10, 100	(x)
3	Rounding: $2^{+}D \times 2^{+}D$ (WN)	(x)			
			7	Mental Computation Compatible Numbers	(x)
			8	Relationship between digits in factors and digits in product	(x)
4	FRONT-END $1D \times 3^{+}D$, $2D \times 2^{+}D$ (WN)	(x)			
			9	Mental Computation 1D x 3D	(x)
5	COMPATIBLE NUMBERS 1D, 2D Divisors (WN)	(÷)			
			10	Mental Computation Multiply by 50, 25	(x)
			11	Find Number of Digits in Quotient 1D, 2D divisors	(÷)
6	COMPATIBLE NUMBERS $\frac{1}{2} \times a \times b$, $\frac{a \times b}{c}$	(x, ÷)			
			12	Number of digits in answer	(+,x,÷)

SCOPE AND SEQUENCE

GRADE 8

LESSON	TOPIC/STRATEGY		MINI LESSON	TOPIC/STRATEGY	
			13	Mental Computation Multiply by powers of 10 (WN, DC)	(x)
7	COMPATIBLE NUMBERS Powers of ten (WN, DC)	(x,÷)			
			14	Identify Fractions Close to $\frac{1}{2}$ and 1	
			15	Sums Greater Than, Less Than 1 (FR)	(+)
			16	Sums Close to 1 and 2 (FR)	(+)
8	FRONT-END, ROUNDING (FR, DC)	(+,x)			
			17	Fraction-Decimal Equivalents	
			18	Find Factional Part $\frac{1}{b} \times c$, $\frac{a}{b} \times c$	(x)
9	COMPATIBLE NUMBERS $\frac{a}{b} \times c$ (FR)	(x,÷)			
			19	Select Fractions Whose Sum/Product is Clolse to a Given Amount	(+,x)
			20	Decimal Approximation for Fractions	
			21	Place Decimal Point in Product (DC)	(x)
			22	Fraction - Percent Equivalents	
10	COMPATIBLE NUMBERS Percent of a Number	(x)			
			23	Find 15% Tip	(x)

APPENDIX B

Attitude Questionnaire and Results

NSF Estimation Project
Attitude Survey

Name: ______________________________

Grade: ______ Teacher: ______________

School: _____________________________

When you estimate answers in arithmetic you get an answer which tells about how much rather than exactly how huch.

> Answer each question below by circling one of the three choices.

ESTIMATION IS SOMETHING:

1. I think is very important.	yes	no	not sure
2. I use outside of school.	yes	no	not sure
3. I use in math class.	yes	no	not sure
4. I like doing.	yes	no	not sure
5. I have been taught in school.	yes	no	not sure
6. I have learned mostly on my own.	yes	no	not sure
7. I am good at doing.	yes	no	not sure
8. I think is hard.	yes	no	not sure

ATTITUDE -- Grade 6, 7, and 8 -- Pre/Post

Estimation is something:

1. I think is very important

		Yes	No	Not Sure
Gr. 6	T	73/79	9/8	18/13
	C	66/63	9/9	25/27
Gr. 7	T	63/62	15/19	22/19
	C	67/61	12/8	21/31
Gr. 8	T	64/75	10/9	26/16
	C	63/73	14/8	24/18

2. I use outside of school

		Yes	No	Not Sure
Gr. 6	T	49/66	37/23	13/9
	C	54/63	34/26	12/10
Gr. 7	T	61/69	29/24	10/5
	C	69/82	23/12	8/6
Gr. 8	T	63/74	27/17	11/9
	C	65/71	24/22	11/6

3. I use in math class

		Yes	No	Not Sure
Gr. 6	T	85/93	7/5	8/2
	C	81/86	13/8	6/5
Gr. 7	T	83/88	8/7	9/2
	C	83/91	7/7	9/2
Gr. 8	T	89/89	5/8	5/2
	C	84/80	9/12	7/8

4. I like doing

		Yes	No	Not Sure
Gr. 6	T	59/34	18/27	23/39
	C	52/42	28/36	20/21
Gr. 7	T	52/25	24/51	22/22
	C	46/42	26/38	28/20
Gr. 8	T	46/35	28/33	25/30
	C	48/40	20/37	31/23

5. I have been taught in school

		Yes	No	Not Sure
Gr. 6	T	87/92	7/3	6/4
	C	88/87	8/9	4/3
Gr. 7	T	91/88	6/9	2/3
	C	93/93	3/3	3/4
Gr. 8	T	90/93	5/4	5/2
	C	93/90	4/4	3/6

6. I have learned mostly on my own

		Yes	No	Not Sure
Gr. 6	T	22/21	59/61	20/18
	C	30/24	49/55	21/20
Gr. 7	T	21/22	67/60	12/17
	C	24/21	57/58	19/20
Gr. 8	T	14/21	69/66	16/11
	C	22/29	64/54	14/16

7. I am good at doing

		Yes	No	Not Sure
Gr. 6	T	54/41	11/13	35/46
	C	53/54	11/8	35/38
Gr. 7	T	52/32	10/23	37/42
	C	50/51	7/13	43/36
Gr. 8	T	46/36	11/18	42/45
	C	56/40	7/16	36/44

8. I think is hard

		Yes	No	Not Sure
Gr. 6	T	12/9	74/70	14/19
	C	6/4	78/78	15/17
Gr. 7	T	9/12	78/66	13/18
	C	5/4	75/87	19/9
Gr. 8	T	9/8	75/74	16/18
	C	5/6	86/78	10/16

APPENDIX C

Mental Computation Tests and Results

Mental Computation

Number: 1

Item:

30 + 40 + 10 + 20

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.94	.91	.90	.93	.91	.90
C	.85	.89	.95	.96	.94	.95

Number: 2

Item:

30 + 400 + 60

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.87	.81	.82	.81	.88	.78
C	.83	.79	.88	.90	.90	.84

Number: 3

Item:

76 + 29

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.66	.49	.53	.55	.73	.62
C	.64	.51	.72	.62	.78	.65

Number: 4

Item:

235 + 60

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.89	.65	.66	.75	.88	.70
C	.79	.70	.89	.80	.88	.79

Mental Computation

Number: 5

Item:

76 + 15

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.80	.61	.63	.65	.85	.61
C	.77	.63	.83	.72	.84	.75

Number: 6

Item:

547 + 199

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.51	.33	.35	.39	.54	.43
C	.43	.35	.54	.42	.51	.51

Number: 7

Item:

357 + 400

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.73	.66	.66	.66	.85	.72
C	.66	.61	.86	.82	.88	.84

Number: 8

Item:

91 - 40

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.53	.45	.45	.51	.64	.54
C	.48	.48	.68	.64	.73	.69

Mental Computation

Number: 9

Item:

86 - 9

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.27	.39	.38	.46	.50	.49
C	.44	.43	.52	.58	.55	.56

Number: 10

Item:

56 - 25

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.37	.41	.35	.50	.51	.52
C	.38	.41	.60	.60	.57	.68

Number: 11

Item:

90 - 32

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.22	.25	.25	.40	.37	.45
C	.24	.35	.43	.51	.53	.60

Number: 12

Item:

150 - 75

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.21	.37	.31	.47	.34	.58
C	.22	.34	.38	.52	.53	.65

Number: 13

Item:

765 - 99

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.06	.08	.04	.15	.15	.25
C	.07	.11	.11	.17	.16	.24

Number: 14

Item:

80 + 40 - 10 + 20 + 30

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.06	.18	.12	.30	.13	.28
C	.09	.21	.20	.28	.23	.39

Number: 15

Item:

65 + 15 - 10 + 25 - 35

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.03	.07	.04	.15	.04	.11
C	.02	.07	.09	.13	.14	.26

Number: 16

Item:

9 x 70

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.65	.78	.75	.80	.86	.85
C	.70	.78	.78	.85	.82	.84

Number: 17

Item:

125 x 10

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.51	.75	.66	.82	.86	.91
C	.60	.70	.72	.83	.80	.89

Number: 18

Item:

400 x 5

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.72	.82	.83	.91	.82	.92
C	.73	.75	.86	.89	.90	.90

Number: 19

Item:

40 x 60

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.52	.74	.55	.81	.77	.89
C	.54	.70	.63	.76	.80	.78

Number: 20

Item:

60 x 15

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.16	.25	.17	.43	.37	.42
C	.14	.27	.30	.39	.45	.48

Number: 21

Item:

962 x 100

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.28	.53	.47	.66	.68	.77
C	.41	.48	.53	.64	.68	.76

Number: 22

Item:

3 x 49

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.26	.35	.36	.52	.58	.45
C	.31	.39	.44	.57	.57	.61

Number: 23

Item:

5 x 99

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.23	.30	.29	.52	.49	.44
C	.29	.39	.43	.51	.59	.56

Number: 24

Item:

300 ÷ 5

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.24	.62	.34	.78	.57	.86
C	.31	.53	.52	.70	.73	.88

Mental Computation

Number: 25

Item:

816 ÷ 4

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.03	.10	.06	.27	.14	.24
C	.05	.15	.17	.33	.29	.38

Number: 26

Item:

400 ÷ 80

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.05	.23	.11	.29	.20	.31
C	.11	.21	.17	.42	.27	.48

Number: 27

Item:

2400 ÷ 60

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.09	.35	.14	.43	.28	.51
C	.13	.30	.17	.42	.37	.56

Number: 28

Item:

200 ÷ 10

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.15	.48	.23	.61	.39	.74
C	.25	.46	.36	.64	.53	.74

Number: 29

Item:

150 ÷ 25

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.04	.25	.11	.30	.23	.44
C	.12	.19	.23	.49	.31	.50

Number: 30

Item:

80,000 ÷ 1,000

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.02	.20	.13	.34	.18	.39
C	.10	.17	.22	.40	.29	.49

Number: 7-31
8-31

Item:

1/2 + 1/4

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.25	.49	.28	.39
C			.30	.41	.28	.49

Number: 7-32
8-32

Item:

1/10 + 1/100

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.14	.37	.26	.24
C			.21	.28	.24	.33

Number: 7-33
8-33

Item:

2 1/5 + 3/5

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.48	.66	.54	.61
C			.56	.60	.57	.68

Number: 7-34
8-34

Item:

3 1/9 + 5

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.30	.48	.32	.48
C			.41	.41	.35	.44

Number: 7-35
8-35

Item:

4 1/2 + 2 1/8

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.11	.32	.17	.21
C			.19	.23	.21	.26

Number: 7-36
8-36

Item:

5/7 - 2/7

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.66	.81	.71	.81
C			.70	.78	.76	.75

Number: 7-37
8-37

Item:

1 - 7/8

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.18	.46	.22	.45
C			.30	.44	.31	.46

Number: 7-38
8-38

Item:

14 - 3/10

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.07	.28	.10	.30
C			.12	.25	.18	.29

Number: 7-39
8-39

Item:

5 3/5 - 2 1/5

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.27	.58	.40	.54
C			.41	.54	.35	.48

Number: 7-40
8-40

Item:

6 - 4 1/2

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.04	.27	.10	.28
C			.11	.20	.16	.22

Mental Computation

Number: 7-41
8-41

Item:

1/2 of 60 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.48	.66	.51	.74
C			.50	.64	.64	.67

Number: 7-42
8-42

Item:

2/3 of 90 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.17	.37	.20	.47
C			.16	.28	.28	.31

Number: 7-43
8-43

Item:

5/4 x 100

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.02	.08	.02	.15
C			.06	.11	.05	.13

Number: 7-44
8-44

Item:

4 x 3 1/2

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			-0-	.05	-0-	.11
C			.03	.07	.05	.08

Mental Computation

Number: 7-45
8-45

Item:

1/100 x 96,000

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			-0-	.05	.02	.10
C			.03	.09	.05	.10

Number: 7-46
8-46

Item:

7 + 4.2

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.43	.56	.51	.56
C			.52	.60	.49	.59

Number: 7-47
8-47

Item:

.3 + .4 + .5

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.34	.52	.51	.58
C			.42	.52	.41	.54

Number: 7-48
8-48

Item:

.56 + .2

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.16	.38	.30	.46
C			.24	.41	.32	.48

Number: 7-49
8-49

Item:

6.28 + 1.99

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.35	.38	.47	.39
C			.48	.52	.53	.45

Number: 7-50
8-50

Item:

1.2 - .4

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.38	.52	.53	.62
C			.45	.58	.42	.66

Number: 7-51
8-51

Item:

9.8 - .5

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.51	.60	.57	.73
C			.63	.70	.59	.69

Number: 7-52
8-52

Item:

80 - .01

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.05	.13	.10	.20
C			.11	.17	.19	.20

Number: 7-53
8-53

Item:

26.0 - .99

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.04	.14	.08	.20
C			.09	.17	.11	.20

Number: 7-54
8-54

Item:

8 x .6

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.25	.52	.40	.66
C			.30	.60	.49	.67

Number: 7-55
8-55

Item:

.8 x 90

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.12	.34	.34	.53
C			.19	.49	.31	.52

Number: 7-56
8-56

Item:

.25 x 80

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.02	.14	.09	.21
C			.05	.14	.10	.21

Number: 7-57
8-57

Item:

85.7 x .1

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.03	.20	.16	.26
C			.06	.28	.16	.28

Number: 7-58
8-58

Item:

4.8 ÷ 1.2

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.02	.09	.01	.14
C			.01	.07	.02	.07

Number: 7-59
8-59

Item:

$.4\overline{).36}$

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.10	.24	.10	.17
C			.15	.24	.12	.21

Number: 7-60
8-60

Item:

200 ÷ .1

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.01	.04	.02	.03
C			.01	.04	.01	.03

Number: 7-61
8-61

Item:

50% of 60 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.41	.66	.58	.83
C			.50	.62	.60	.69

Number: 7-62
8-62

Item:

10% of 26 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.06	.17	.14	.31
C			.08	.21	.14	.33

Number: 7-63
8-63

Item:

25% of 8 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.35	.61	.43	.72
C			.40	.50	.52	.67

Number: 7-64
8-64

Item:

75% of 200 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.18	.29	.28	.49
C			.20	.34	.24	.45

Mental Computation

Number: 7-65
8-65

Item:

40% of 80 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			-0-	.04	.06	.14
C			.03	.09	.05	.11

Number: 7-66
8-66

Item:

100% of 9 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.42	.59	.50	.75
C			.46	.60	.52	.67

Number: 7-67
8-67

Item:

150% of 300 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.05	.09	.12	.25
C			.09	.15	.16	.23

Number: 7-68
8-68

Item:

.1% of 6000 is

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.05	.06	.04	.13
C			.02	.11	.05	.18

APPENDIX D

Computational Estimation Test, Acceptable Intervals and Results

Estimation Test

Number: 6-1-C
7-1-C
8-1-C

Item:

3596 + 6125

Acceptable Interval:

9,500 - 10,000

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.50	.59	.55	.67	.64	.66
C	.47	.59	.62	.65	.63	.71

Number: 6-2-C
7-2-C
8-2-C

Item:

147 + 561 + 85

Acceptable Interval:

700 -850

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.46	.54	.51	.67	.53	.72
C	.38	.53	.50	.68	.67	.73

Number: 6-3-C
7-3-C
8-3-C

Item:

```
  8419
  9275
  9015
  8191
+ 9989
```

Acceptable Interval:

43,000 - 46,000

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.08	.25	.08	.35	.18	.42
C	.12	.22	.17	.17	.18	.29

Number: 6-4-C
7-4-C
8-4-C

Item:

```
  35216
   4912
  15475
+   317
```

Acceptable Interval:

52,000 - 57,000

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.11	.36	.21	.42	.26	.48
C	.15	.31	.28	.28	.29	.38

Number: 6-5-C
7-5-C
8-5-C

Item:

2888 - 979

Acceptable Interval:

1,800 - 2,000

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.31	.46	.39	.52	.42	.55
C	.35	.48	.44	.42	.56	.63

Number: 6-6-C
7-6-C
8-6-C

Item:

37689 - 18812

Acceptable Interval:

18,000 - 20,000

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.16	.46	.24	.57	.36	.63
C	.22	.37	.29	.40	.35	.53

Number: 6-7-C
7-7-C
8-7-C

Item:

28 x 47

Acceptable Interval:

1,200 - 1,500

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.08	.36	.20	.38	.24	.60
C	.14	.42	.26	.34	.37	.46

Number: 6-8-C
7-8-C
8-8-C

Item:

427 x 8

Acceptable Interval:

3,200 - 3,600

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.39	.70	.51	.59	.58	.63
C	.43	.51	.52	.59	.59	.64

Number: 6-9-C
7-9-C
8-9-C

Item:

104 x 36

Acceptable Interval:

3,600 - 4,000

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.11	.44	.26	.48	.41	.63
C	.22	.42	.33	.42	.38	.52

Number: 6-10-C
7-10-C
8-10-C

Item:

557 ÷ 8

Acceptable Interval:

60 - 70

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.11	.50	.20	.51	.41	.66
C	.19	.33	.28	.40	.48	.61

Number: 6-11-C
7-11-C
8-11-C

Item:

$32\overline{)947}$

Acceptable Interval:

20 - 30

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.12	.44	.34	.54	.40	.67
C	.19	.41	.30	.45	.43	.53

Number: 6-12-C
7-12-C
8-12-C

Item:

6548 ÷ 96

Acceptable Interval:

64 - 70

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.05	.18	.10	.26	.19	.36
C	.10	.21	.15	.33	.32	.35

Number: 6-13-C
7-13-C
8-13-C

Item:

8127 $\overline{)474,257}$

Acceptable Interval:

50 - 60

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.01	.04	.04	.09	.08	.16
C	.01	.02	.03	.07	.07	.10

Number: 6-14-C
7-14-C
8-14-C

Item:

$$\frac{217 \times 4}{17}$$

Acceptable Interval:

40 - 55

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.03	.11	.03	.20	.14	.46
C	.01	.12	.03	.16	.20	.31

Number: 6-15-C
7-15-C
8-15-C

Item:

2.49 + 16.19 + .08 + 1.27

Acceptable Interval:

19 - 21

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.18	.54	.30	.63	.31	.72
C	.21	.36	.38	.52	.44	.59

Number: 6-16-C

Item:

8.18 - 6.97

Acceptable Interval:

1 - 1.5

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.23	.42				
C	.26	.33				

Number: 7-16-C
8-16-C

Item:

349.1 + .0097 + 19.37

Acceptable Interval:

360 - 370

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.16	.42	.19	.49
C			.15	.32	.20	.33

Number: 6-17-C
7-17-C
8-17-C

Item:

427 x 9.8

Acceptable Interval:

3,800 - 4,300

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.03	.23	.11	.32	.22	.52
C	.08	.24	.15	.25	.17	.35

Number: 6-18-C
7-18-C
8-18-C

Item:

41 x .75

Acceptable Interval:

28 - 32

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.02	.08	.07	.12	.10	.20
C	.04	.10	.03	.12	.12	.16

Number: 6-19-C
7-21-C
8-21-C

Item:

$\frac{12}{13} + \frac{7}{8}$

Acceptable Interval:

1½ - 2

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.03	.11	.04	.16	.02	.32
C	.01	.08	.02	.14	.10	.17

Number: 6-20-C
7-22-C
8-22-C

Item:

$3\frac{1}{5} + 2\frac{5}{7} + \frac{9}{10}$

Acceptable Interval:

6 - 7

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.04	.28	.09	.35	.19	.58
C	.10	.21	.23	.30	.24	.35

Number: 7-19-C
8-19-C

Item:

$83\,/\overline{3\ 9}$

Acceptable Interval:

.4 - .5

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.04	.17	.14	.34
C			.11	.17	.15	.36

Number: 7-20-C
8-20-C

Item:

376 ÷ .98

Acceptable Interval:

370 - 400

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.07	.18	.17	.41
C			.14	.20	.17	.26

Number: 7-23-C
8-23-C

Item:

$\frac{9}{10} \times 14\frac{1}{3}$

Acceptable Interval:

12 - 14 1/3

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.28	.35	.24	.42
C			.32	.34	.33	.29

Number: 7-24-C
8-24-C

Item:

$3\frac{2}{5} \times 7\frac{1}{8}$

Acceptable Interval:

21 - 28

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.36	.52	.46	.60
C			.40	.52	.46	.48

Number: 7-25-C
8-25-C

Item:

$216\frac{1}{8} \div 9\frac{5}{6}$

Acceptable Interval:

20 - 24

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.07	.24	.14	.36
C			.09	.28	.18	.23

Number: 7-26-C
8-26-C

Item:

23% of 42

Acceptable Interval:

8 - 11

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.09	.23	.20	.40
C			.17	.19	.14	.22

Number: 7-27-C
8-27-C

Item:

98% of 114

Acceptable Interval:

108 - 114

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.19	.35	.28	.47
C			.28	.41	.29	.39

Estimation Test

Number: 7-28-C
8-28-C

Item:

15% of 23.19

Acceptable Interval:

3 - 4

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.04	.09	.09	.20
C			.05	.15	.03	.10

Number: 7-29-C
8-29-C

Item:

49% of 118

Acceptable Interval:

55 - 60

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.20	.40	.25	.58
C			.27	.35	.24	.36

Number: 7-30-C
8-30-C

Item:

32% of 61

Acceptable Interval:

18 - 21

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.12	.27	.19	.42
C			.17	.26	.22	.39

Estimation Test

Number: 6-1-A

ABOUT how much does this 3 piece suit cost?

$29.95	vest
$47.95	slacks
$98.95	jacket

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.20	.46				
C	.26	.43				

Acceptable Interval:

170 - 180 (176.85)

Number: 6-2-A
7-1-A
8-1-A

ESTIMATE the TOTAL enrollment in these schools.

	ENROLLMENT
BLUE EYE	398
CAMEL BACK	1506
DOG LEG	218
EAGLE NEST	23
FOX HOLE	3115

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.13	.32	.25	.40	.23	.42
C	.16	.21	.27	.32	.32	.39

Acceptable Interval:

5,000 - 5,500 (5260)

Number: 6-3-A
7-2-A
8-2-A

WORLD'S FAIR ATTENDANCE
JUNE 1-7

Sunday	84,326
Monday	72,519
Tuesday	77,942
Wednesday	81,419
Thursday	75,569
Friday	77,602
Saturday	87,363

ABOUT how many people attended the fair this week ?

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.01	.07	.01	.05	.02	.13
C	.01	.01	.03	.03	.02	.05

Acceptable Interval:

520,000 - 570,000 (556,740)

Estimation Test

Number: 6-4-A
7-3-A
8-3-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.28	.44	.37	.48	.54	.63
C	.33	.47	.50	.49	.56	.52

ABOUT how much will these 6 sandwiches cost?

BEEF	$2.89	HAM	$3.15
BLT	$2.95	RUEBEN	$3.19
SPECIAL	$2.79	POOR BOY	$2.99

Acceptable Interval:

$16 - $18.50 ($17.96)

Number: 6-5-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.19	.44				
C	.22	.26				

ABOUT how many calories did I take in today.

BREAKFAST	608
SNACK	405
LUNCH	788
SNACK	289
DINNER	820
SNACK	149

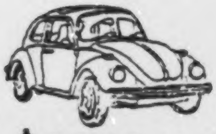
Acceptable Interval:

2,800 - 3,100 (3,059)

Number: 6-6-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.30	.56				
C	.37	.48				

ABOUT what is the difference in price?



$3,788

$12,367

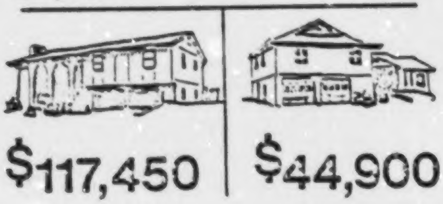
Acceptable Interval:

8,000 - 9,000 (8,579)

Estimation Test

Number: 6-7-A

ABOUT what is the difference in price?



	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.12	.24				
C	.12	.12				

Acceptable Interval:

70,000 - 80,000 (72,550)

Number: 6-8-A
7-8-A
8-4-A

We deliver 290 papers a day.
ABOUT how many papers in a week?

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.23	.46	.34	.49	.37	.52
C	.27	.33	.30	.38	.47	.50

Acceptable Interval:

1,800 - 2,100 (2,030)

Number: 6-9-A

We deliver 107 papers a day.
ABOUT how many papers this month?

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.12	.37				
C	.13	.25				

Acceptable Interval:

3,000 - 3,500 (3,210 OR 3,317)

Estimation Test

Number: 6-10-A
7-9-A
8-5-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.05	.14	.04	.17	.16	.23
C	.10	.15	.13	.11	.16	.17

We deliver 95 papers a day.
About how many papers in a year ?

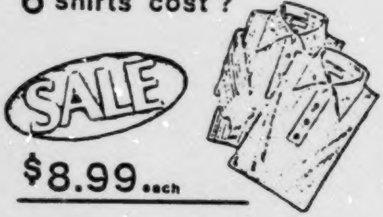
Acceptable Interval:

32,000 - 37,000 (34,675)

Number: 6-11-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.27	.47				
C	.31	.43				

ABOUT how much will 6 shirts cost ?



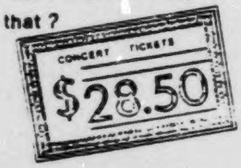
Acceptable Interval:

52 - 57 (53.94)

Number: 6-12-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.06	.18				
C	.09	.17				

78 tickets were sold immediately.
ABOUT how much money is that ?



Acceptable Interval:

2,100 - 2,400 (2,223)

Estimation Test

Number: 6-13-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.13	.37				
C	.17	.30				

ABOUT how much will 9 umbrella's cost ?

Acceptable Interval:

$150 - $180 ($161.55)

Number: 6-14-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.06	.29				
C	.05	.11				

An 8-pack of soda costs $3.38
ABOUT how much does 1 bottle cost?

Acceptable Interval:

40¢ - 45¢ (42.25¢)

Number: 6-15-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.02	.18				
C	.07	.12				

George has a car loan of $9,875
to repay in 48 months.
ESTIMATE his monthly payment.

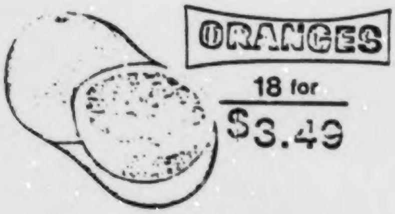
Acceptable Interval:

$200 - $250 ($205.73)

Number: 6-16-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.13	.20				
C	.12	.26				

ABOUT how much does each orange cost?



Acceptable Interval:

15¢ - 20¢ (19.39¢)

Number: 6-17-A
7-12-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.02	.18	.05	.24		
C	.02	.14	.09	.15		

Attendance at the first 4 nights of the show was 1749.
ABOUT what was the average attendance?

Acceptable Interval:

400 - 450 (437.25)

Number: 6-18-A
7-16-A
8-9-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.01	.19	.09	.29	.16	.37
C	.06	.18	.11	.18	.20	.23

This glider traveled 6153 miles on 19 gallons of fuel.
ABOUT how many miles per gallon?

Acceptable Interval:

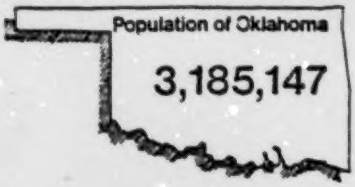
300 - 350 (323.84)

Estimation Test

Number: 6-19-A
7-15-A
8-10-A

Oklahoma has 6 representatives in congress.

ABOUT how many people does each representative represent ?



	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.01	.18	.08	.20	.16	.37
C	.05	.12	.05	.15	.16	.29

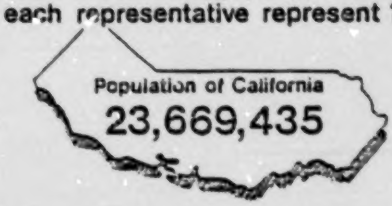
Acceptable Interval:

500,000 - 550,000 (530,857.83)

Number: 6-20-A

California has 43 representatives in congress.

ABOUT how many people does each representative represent ?



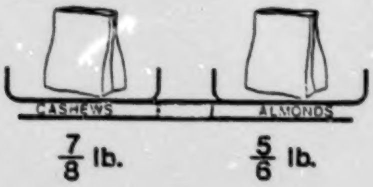
	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T	.03	.10				
C	.06	.08				

Acceptable Interval:

500,000 - 600,000 (550,451.97)

Number: 7-4-A

ABOUT how much will these nuts weigh altogether.


CASHEWS
ALMONDS
7/8 lb.
5/6 lb.

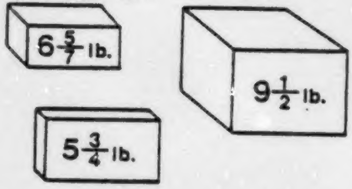
	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.11	.40		
C			.13	.31		

Acceptable Interval:

1½ - 2 (1.71)

Number: 7-5-A

ABOUT how much will these 3 boxes weigh altogether?


$6\frac{5}{7}$ lb.
$9\frac{1}{2}$ lb.
$5\frac{3}{4}$ lb.

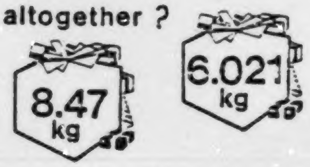
	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.29	.48		
C			.36	.48		

Acceptable Interval:

21 - 23 (21.96)

Number: 7-6-A

ABOUT how much do these boxes weigh altogether ?



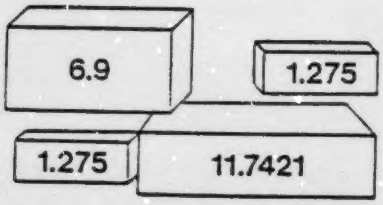
	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.43	.70		
C			.65	.74		

Acceptable Interval:

14 -15 (14.49)

Number: 7-7-A

ABOUT how much do these boxes weigh altogether?



	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.21	.54		
C			.40	.44		

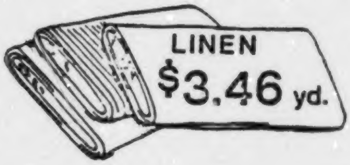
Acceptable Interval:

20 - 22 (21.19)

Estimation Test

Number: 7-10-A

ABOUT how much will $4\frac{3}{4}$ yards cost?



	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.16	.34		
C			.25	.29		

Acceptable Interval:

14 - 19 (16.44)

Number: 7-11-A

We have $3\frac{1}{2}$ rolls of wallpaper left. Each roll gives us $6\frac{1}{4}$ strips for our wall.

ABOUT how many strips of paper is that?

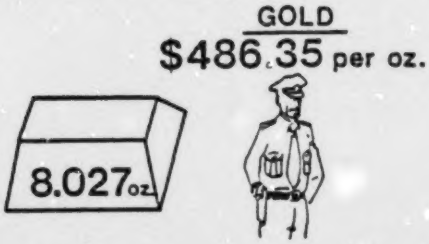
	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.05	.12		
C			.08	.19		

Acceptable Interval:

19 - 24 (21.875 OR 21 7/8)

Number: 7-13-A
8-8-A

ABOUT how much will this bar be worth.


GOLD
$486.35 per oz.
8.027oz.

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.04	.22	.07	.21
C			.07	.15	.10	.18

Acceptable Interval:

3,500 - 4,000 (3,903.93)

Number: 7-14-A
8-7-A

ABOUT how much will a load of 98 suitcases weigh?

Average suitcase weight 21.69 kg

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.08	.22	.20	.38
C			.15	.22	.14	.26

Acceptable Interval:

2,000 - 2,200 (2125.62)

Number: 7-17-A
8-12-A

Jen made ABOUT $\frac{1}{3}$ of her shots.
Jen took 76 shots.
ABOUT how many shots did she make?

______

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.15	.25	.28	.41
C			.22	.27	.19	.37

Acceptable Interval:

24 - 27 (25 1/3)

Number: 7-18-A

I have about $\frac{1}{4}$ of my 463 page book to read.
ABOUT how many pages is that?

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.20	.32		
C			.25	.28		

Acceptable Interval:

105 - 125 (115.75 OR 115 3/4)

Estimation Test

Number: 7-19-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.11	.21		
C			.17	.14		

ESTIMATE the price per pound.

6.24 lbs. **$45.85**

Caviar

Acceptable Interval:

7 - 8 (7.35)

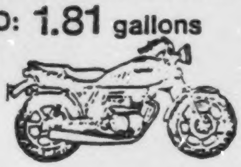
Number: 7-20-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T			.02	.09		
C			.05	.08		

ABOUT how many miles per gallon is the cycle getting ?

TRAVELED: 132.4 miles

USED: 1.81 gallons



Acceptable Interval:

60 - 80 (73.15)

Number: 8-6-A

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					.20	.26
C					.15	.20

Each person will spend about $12 for the game.

ABOUT how much money will be collected ?

ATTENDANCE

98,654

Acceptable Interval:

$980,000 - $1.2 MILLION

($1.18 MILLION)

Estimation Test

Number: 8-11-A

ESTIMATE the price per ounce.

9.981 oz GOLD $4779.76

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					.09	.29
C					.08	.20

Acceptable Interval:

$470 - $500 ($478.89)

Number: 8-13-A

ABOUT one-tenth of school children wear glasses.

ABOUT how many children wear glasses?

School Population

32,469,000

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					.01	.11
C					.03	.07

Acceptable Interval:

3.2 - 3.3 MILLION

(3.2469 MILLION)

Number: 8-14-A

We want 9 pounds and will split the cost 3 ways. ABOUT how much will it cost each of us?

PORK STEAK $1.19 lb.

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					.30	.41
C					.39	.50

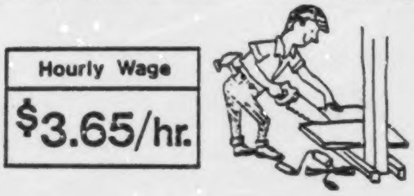
Acceptable Interval:

$3 - $4 ($3.57)

Estimation Test

Number: 8-15-A

I worked about 20 hours and plan to save $\frac{1}{3}$ of my earnings. ABOUT how much will be saved?



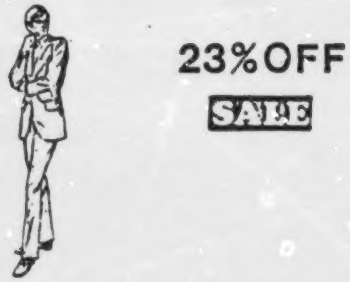
	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					.15	.34
C					.14	.24

Acceptable Interval:

20 - 27 (24 1/3)

Number: 8-16-A

ESTIMATE the savings on a suit priced at $79.95.



	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					.08	.27
C					.14	.17

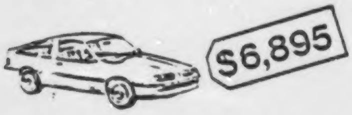
Acceptable Interval:

16 - 20 (18.39)

Number: 8-17-A

All cars-reduced 30%

ABOUT how much will this car price be reduced?



	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					.11	.24
C					.06	.15

Acceptable Interval:

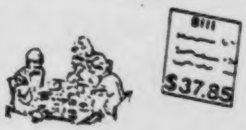
2,000 - 2,300 (2,068.5)

Estimation Test

Number: 8-18-A

ABOUT how much should be left for the tip.

15% tip requested



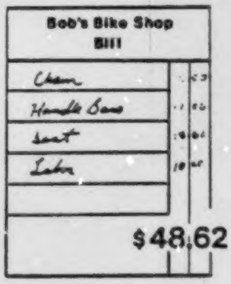
	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					.13	.20
C					.13	.15

Acceptable Interval:

4.50 - 6.25 (5.68)

Number: 8-19-A

The tax rate is 4¾%. ESTIMATE the tax.



	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					.07	.12
C					.04	.12

Acceptable Interval:

2.00 - 2.50 (2.31)

Number: 8-20-A

ABOUT 20% of the population has blue eyes.

ABOUT how many people have blue eyes?

U. S. Population

225,437,263

	6th		7th		8th	
	Pre	Post	Pre	Post	Pre	Post
T					-0-	.11
C					.03	.03

Acceptable Interval:

40 - 50 MILLION

(45.09 MILLION)

APPENDIX E

Midyear Test and Results

MID-YEAR RESULTS

ITEM	RANGE	6th 180	7th 177	8th 200
$1.59 .37 .25 .79 2.95	$4.00 - $6.25	63.3	69.5	80.0
44,159 8,274 + 53,195	98,000 - 110,000	44.4	52.0	73.0
19 + 3847 + 485 + 1995	5300 - 6520	15.0	23.7	44.0
4.172 + 18.1 + 5.7364 + 0.26	27 - 30			37.5

73 74 69 66 67 72 + 71	460 - 511	51.7	45.8	62.0
4,859,864 5,727,639 4,998,278 + 4,632,745	18^+ - 21 million	34.4	38.4	62.5
1875 + 2163 + 2027 + 1875	7000 - 8000	40.0	44.1	
3.27 + 2.975 + 3.087 + 2.8 + 2.7962	13 - 15.5			53.0
$14\frac{1}{9} + 15\frac{1}{3} + 16\frac{1}{7} + 15\frac{3}{4}$	60 - 62			31.5

MID-YEAR RESULTS

ITEM	RANGE	6th	7th	8th
517 - 248	240 - 300	68.9	71.8	
6257 - 489	5500 - 5800	45.6	54.2	
37,245 - 18,307	17,000 - 20,000	43.3	53.1	

8 x 583	4000⁺ - 4800	51.7	51.4	52.0
39 x 68	2500 - 2800	43.9	40.7	61.5
41 x 72	2800 - 3000	58.9	60.0	69.5
6 x 2137	12,000 - 13.000	59.4	71.8	
69 x 4827	300,000 - 350,000		32.2	
98 x 472	45,000 - 50,000			48.5
512 x 321	150,000 - 170,000			55.5

MID-YEAR RESULTS

ITEM		RANGE	6th	7th	8th
$6\,\overline{)4806}$	80 800 8000	800	92.2	90.4	
$3\,\overline{)191}$	6 60 600	60	95.0		
$31\,\overline{)1916}$	60 600 6000	60		81.4	
$7\,\overline{)2746}$	200 300 400	400	51.5	31.1	
$9\,\overline{)43,427}$	4000 5000 6000	5000	47.2	26.6	
$92\,\overline{)43,927}$	400 500 600	500			

$6\,\overline{)3158}$	500 - 550	74.4	83.6	81.0
$4\,\overline{)269}$	60 - 70	73.3	75.7	
$2\,\overline{)1399}$	600 - 700	80.0		
$21\,\overline{)1399}$	60 - 70		49.2	
$39\,\overline{)1597}$	30 - 45		27.1	
$8\,\overline{)26,534}$	3000 - 4000			72.5
$92\,\overline{)735}$	7.5 - 8.5			33.0
$43\,\overline{)812,594}$	18,000 - 20,000			40.0
$82\,\overline{)59,251}$	700 - 800			26.5

ITEM		RANGE	MID-YEAR RESULTS 6th	 7th	 8th
$7.98 + $7.62	more than $15 less than $15 hard to tell	more	89.4	92.1	93.5
92 + 89 + 91 + 93	more than 360 less than 360 hard to tell	more	63.3	59.3	
9624 - 3268	more than 6000 less than 6000 hard to tell	more	52.2	55.9	
24 x 98	more than 2400 less than 2400 hard to tell	less	55.0	51.4	61.0
47 x 29	more than 1500 less than 1500 hard to tell	less	49.4	44.6	
$4\overline{)2745}$	more than 700 less than 700 hard to tell	less	45.0	51.4	65.5
298,466 281,543 + 304,875	more than 900,000 less than 900,000 hard to tell				51.0
$79\overline{)165,269}$	more than 2000 less than 2000 hard to tell				50.5
$\frac{1}{2} \times 854 \times 32$	more than 12,000 less than 12,000 hard to tell				28.0

MID-YEAR RESULTS

ITEM	RANGE	6th	7th	8th
TICKET - $9.35 329 tickets were sold in one day. About how much money was collected?	2700 - 3300	26.1	35.0	
COATS --- $49.88 SHIRTS -- 7.77 TIES --- 2.19 About how much do 2 shirts and a tie cost?	$\$16^{+} - \$19$	50.0	56.5	56.0
About how much do these cost altogether? $ .37 $2.19 $ .59 $ .82	$3.00 - 4.25			84.0
FILM: 24 exposures $4.69 About how much does one picture cost?	18 - 20			33.5
A bus holds 48 students. About how many students can be taken in 24 buses?	1000 - 1250	42.8	39.5	
A school purchased 8 desks at a total cost of $3127. About how much did they pay for each desk?	300 - 400	43.3		
It cost $81,647 to stay in a hospital for 39 days. About how much did it cost per day?	2000 - 2200		27.7	43.0
CAPACITY 18,459 TICKETS $ 9.25 About how much is made from a sellout?	$160,000 - 190,000^{-}$			33.0
A bus holds 48 students. About how many students can be taken in 19 buses?	$800^{+} - 1000$			63.5

MID-YEAR RESULTS

ITEM	RANGE	6th	7th	8th
$\frac{1}{5} \times 34 \times 41$	240 - 320¯			10.0
$\frac{1}{6} \times 17 \times 418$	1150 - 6000¯			10.0
$\frac{1}{3} \times 299 \times 52$	4900 - 6000¯			12.5
$\frac{347 \times 33}{17}$	600 - 700			8.5

On the final page of the Mid-Year Test, students were given Susie's paper which they were told contained some errors. They were asked to find the three problems which were not sensible.

$\begin{array}{r}249\\27\\+\ 416\\\hline 692\end{array}$		81.1	74.6	
$\begin{array}{r}4663\\-167\\\hline 4,496\end{array}$		57.2	64.4	
$\begin{array}{r}98\\\times\ 24\\\hline 23,502\end{array}$	*	71.7	68.4	61.0
$\begin{array}{r}95\\5\,\overline{)475}\end{array}$		81.7		
$\begin{array}{r}247\\\times\ 6\\\hline 4482\end{array}$		66.1		
$\begin{array}{r}22\\6\,\overline{)1320}\end{array}$	*	51.1	50.8	50.0
$6157 + 700 + 3478 = 16,635$	*	54.4	62.7	69.0

APPENDIX F

Summary of Interview Packet

To Student: I am interviewing students to find how you make estimates. I will show you a few problems and I would like for you to estimate the answer to each. As you estimate, I want you to tell me what you are thinking. This will help me understand how you get your estimate. You may not think some of the things are important but they may help me understand what you are thinking, so please think out aloud. Do you understand?

At the close of interview

"Where did you learn to estimate? (i.e. Who taught you?)"

"What kind of estimation problems are hardest for you?"

"What are some hints you would give someone that wanted to learn how to estimate?"

(In April only) "What are some of the most important things you learned this year about estimation?"

NSF ESTIMATION PROJECT

Interview Summary Sheet - Grade 6

Name: ______________________

School: ______________________

Teacher: ______________ Group: __________

PROBLEM	ESTIMATE	STRATEGY			
#1 37 419 46 785 41 045 + 38 974		Rounded to: 10,000 1,000 100	Front-End	Average	Adjustment

PROBE: Above or Below? Why?

NOTES:

#2 729 - 371		Rounded to: 100 10	Front-End	Adjustment

PROBE: Over or Under?

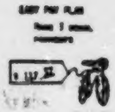
NOTES:

#3 2548 ÷ 43		Round	Compatible Numbers	Multiplication Guess-Check	Adjustment

PROBE: More than 50?

NOTES:

#4



Round	Compatible Numbers	Multiplication Guess-Check	Adjustment

NOTES:

#5 $\frac{12}{13} + \frac{7}{8}$		Add numerators and denominators	Recognize Fractions Near One	Adjustment

PROBE: Above or Below?

NOTES:

#6

BLUE EYE	[illegible]
CAMEL BACK	[illegible]

[illegible]

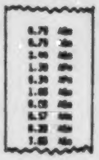
Rounded to: 10,000 1,000 100	Rounded to Compatible Numbers	Front-End	Adjustment

PROBE: Another way?

NOTES:

PROBLEM ESTIMATE STRATEGY

#7



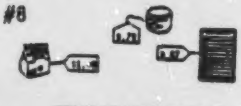
Rounded to: Compatible Numbers (Grouping) Front-End Adjustmen

Dollar Dime

PROBE: How close?

NOTES:

#8

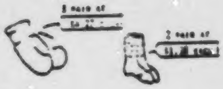


Add total then subtract | Subtract Indiv. Items from $5 | Round dollar dime | Front-End | Compatible Numbers

PROBE: More than $1? $2? $3?

NOTES:

#9



Rounded to: Dollar Dime | Compatible Numbers | Front-End | Adjustment

PROBE: Could you buy them for $37?
Could you buy them for (Response)?

NOTES:

#10

Reasonable? YES NO | Rounded to: 20x40 20x50 30x50 | Checked ones place? | Adjustment

PROBE: How did you know?

NOTES:

Where did you learn to estimate?

What kind of problems are the hardest?

Hints:

Interviewer Comments:

1.

$$\begin{array}{r} 37\ 419 \\ 46\ 765 \\ 41\ 045 \\ +\ 34\ 974 \\ \hline \end{array}$$

PROBE: DO YOU THINK THE ACTUAL ANSWER IS ABOVE OR BELOW YOUR ESTIMATE?

2.

$$\begin{array}{r} 729 \\ -\ 371 \\ \hline \end{array}$$

PROBE: DO YOU THINK IT IS OVER (400, 300) OR UNDER?

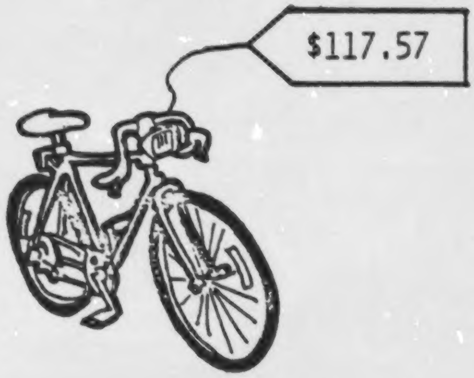
3. 2548 ÷ 43

Probe if child going nowhere: Is it more than 50? Why?

4.

EASY PAY PLAN

Make 3 equal payments



Probe: About how much will each payment cost?

5.

$$\frac{12}{13} + \frac{7}{8}$$

PROBE: DO YOU THINK THE ACTUAL ANSWER IS ABOVE OR BELOW YOUR ESTIMATE?

6.

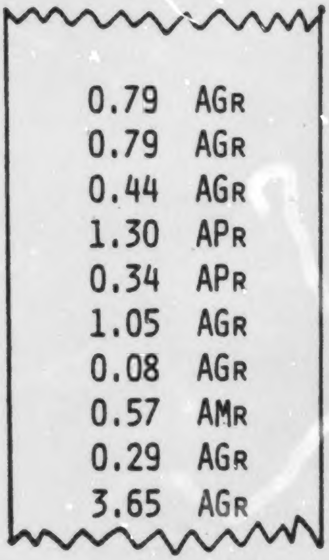
BLUE EYE	47 349
CAMEL BACK	28 563

ABOUT WHAT IS THE DIFFERENCE IN THE POPULATIONS OF THESE TWO TOWNS?

PROBE: CAN YOU DO IT ANOTHER WAY?

7.

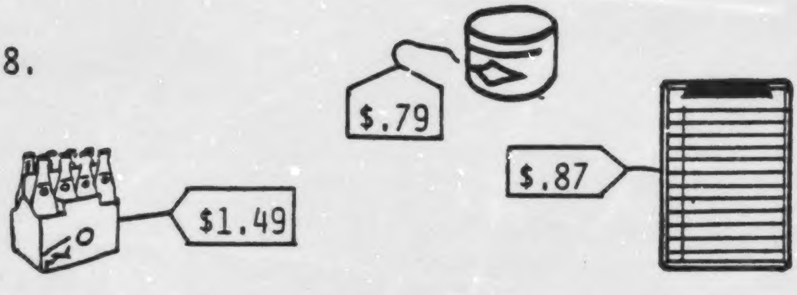
HERE IS A GROCERY TICKET WHICH HAS NOT YET BEEN TOTALED. ESTIMATE THE TOTAL.



0.79	AGR
0.79	AGR
0.44	AGR
1.30	APR
0.34	APR
1.05	AGR
0.08	AGR
0.57	AMR
0.29	AGR
3.65	AGR

PROBE: HOW CLOSE DO YOU THINK YOUR ESTIMATE IS?

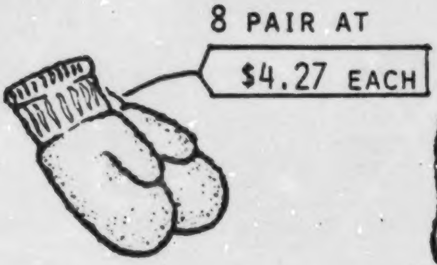
8.

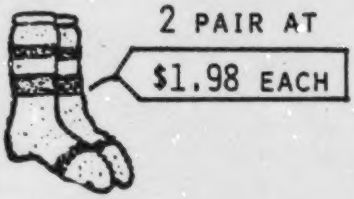


ABOUT HOW MUCH CHANGE WOULD YOU GET FROM $5?

PROBE: MORE THAN $1?
$2?
$3?

9.



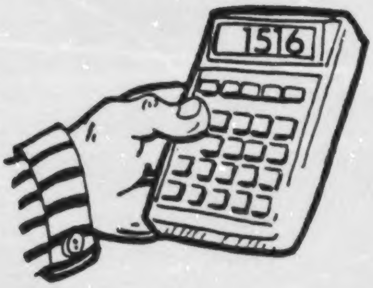


About how much would this cost?

Probe: If RESPONSE < $37, ask Could you buy them for $37?

If RESPONSE > $37, ask Could you buy them for RESPONSE?

10. 22 x 48



Is this calculator result reasonable?

Probe: How did you know?

NSF ESTIMATION PROJECT

Interview Summary Sheet - Grade 7

Name: ____________________

School: ____________________

Teacher: ________ Group: _____

PROBLEM **ESTIMATE** **STRATEGY**

#1

$\begin{array}{r} 37\,419 \\ 46\,765 \\ 41\,045 \\ +\ 34\,974 \\ \hline \end{array}$	Rounded to: 10,000 1,000 100	Front-End	Average	Adjustment

PROBE: Above or Below? Why?

NOTES:

- -

#2

$\frac{12}{13} + \frac{7}{8}$	Add numerators and denominators	Recognize Fractions Near One	Adjustment

PROBE: Above or Below?

NOTES:

- -

#3

		Front-End			
$2\frac{1}{5} + 7\frac{5}{6} + \frac{9}{10}$	Round	whole numbers only	with fractions	Other	Adjustment

PROBE: Estimate more than exact answer?

NOTES:

- -

#4

		Front-End			
342.24 + 8.8 + 229.09 + 2.913	Round	whole numbers only	use of all decimals	Other	Adjustment (Recognition of significant digits

NOTES:

- -

#5

.943 x 8076	Round	Front-End use of algorithm to place decimal	Special Number (near 1)	Adjustment

PROBE: Above or Below?

NOTES:

- -

#6

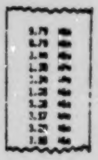
4338 ÷ 93	Round	Compatible Numbers	Special Numbers	Other	Adjustment

PROBE: Another way?

NOTES:

PROBLEM **ESTIMATE** **STRATEGY**

#7



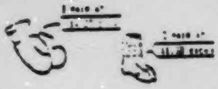
Rounded to: Dollar Dime   Compatible Numbers (Grouping)   Front-End   Adjustment

PROBE: How close?

NOTES:

- -

#8



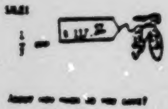
Rounded to: Dollar Dime   Compatible Numbers   Front-End   Adjustment

PROBE: Could you buy them for $37?
Could you buy them for (Response)?

NOTES:

- -

#9



Round to $120   Front-End   Multiplication Guess-Check   Compatible Numbers   Adjustmen

PROBE:

NOTES:

- -

#10

Reasonable? YES NO   Rounded to: 20x40 20x50 30x50   Checked ones place?   Adjustment

PROBE: How did you know?

NOTES:

- -

Where did you learn to estimate?

What kind of problems are the hardest?

Hints:

Interviewer Comments:

1.

$$\begin{array}{r} 37\,419 \\ 46\,765 \\ 41\,045 \\ +\ 34\,974 \\ \hline \end{array}$$

PROBE: DO YOU THINK THE ACTUAL ANSWER IS ABOVE OR
BELOW YOUR ESTIMATE?

2.

$$\frac{12}{13} + \frac{7}{8}$$

PROBE: DO YOU THINK THE ACTUAL ANSWER IS ABOVE OR
BELOW YOUR ESTIMATE?

3.

$$2\frac{3}{5} + 7\frac{5}{6} + \frac{9}{10}$$

PROBE: IS YOUR ESTIMATE MORE THAN THE EXACT ANSWER?

4.

342.24 + 8.8 + 229.09 + 2.913

5.

.943 x 8076

PROBE: DO YOU THINK THE ACTUAL ANSWER IS ABOVE OR BELOW YOUR ESTIMATE?

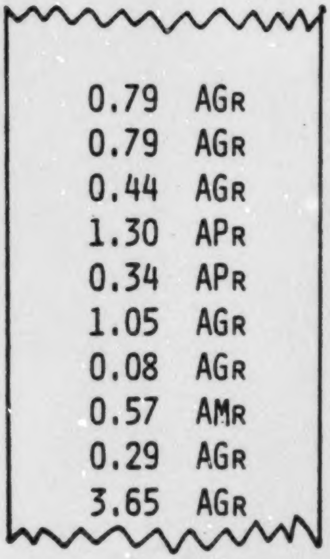
6.

4338 ÷ 93

PROBE: IS THERE ANOTHER WAY YOU COULD DO IT?

7.

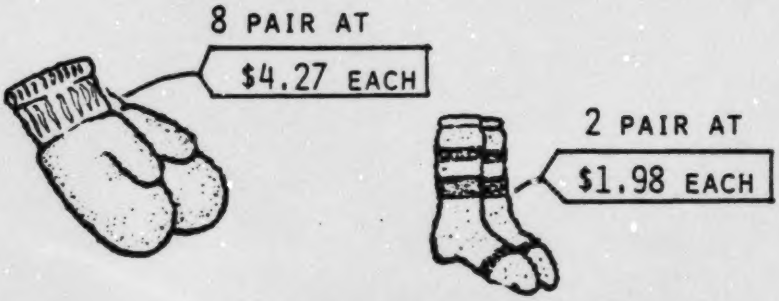
HERE IS A GROCERY TICKET WHICH HAS NOT YET BEEN TOTALED. ESTIMATE THE TOTAL.



0.79	AGR
0.79	AGR
0.44	AGR
1.30	APR
0.34	APR
1.05	AGR
0.08	AGR
0.57	AMR
0.29	AGR
3.65	AGR

PROBE: HOW CLOSE DO YOU THINK YOUR ESTIMATE IS?

8.



ABOUT HOW MUCH WOULD THIS COST?

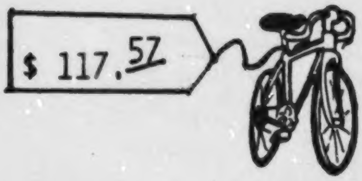
PROBE: IF RESPONSE < $37, ASK COULD YOU BUY THEM FOR $37?

IF RESPONSE > $37, ASK COULD YOU BUY THEM FOR RESPONSE?

9.

SALE!

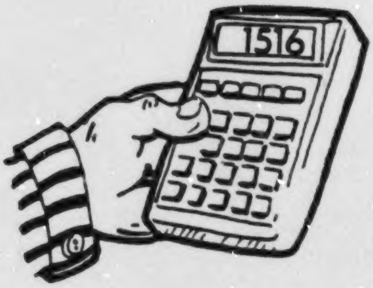
$\frac{1}{3}$ OFF



ABOUT HOW MUCH DO YOU SAVE?

10.

22 x 48



IS THIS CALCULATOR RESULT REASONABLE?

PROBE: HOW DID YOU KNOW?

NSF ESTIMATION PROJECT

Interview Summary Sheet - Grade 8

Name: ____________________

School: ____________________

Teacher: ____________ Group: __________

PROBLEM	ESTIMATE	STRATEGY			
#1 37 419 46 765 21 045 + 38 974		Rounded to: 10,000 1,000 100	Front-End	Average	Adjustment

PROBE: Above or Below? Why?

NOTES:

#2 $\frac{12}{13} + \frac{7}{8}$	Add numerators and denominators	Recognize Fractions Near One	Adjustment

PROBE: Above or Below?

NOTES:

#3 $2\frac{3}{5} + 7\frac{5}{6} + \frac{9}{10}$	Round	Front-End whole numbers only with fractions	Other	Adjustment

PROBE: Estimate more than exact answer?

NOTES:

#4 .983 x 8076	Round	Front-End use of algorithm to place decimal	Special Number (near 1)	Adjustment

PROBE: Above or Below?

NOTES:

#5 $\frac{346 \times 9}{43}$	Rounded multiplied then div. also used 9 as special	Used a format of $\frac{a \times b}{c} = \frac{d}{c}$ $\frac{a \times b}{c} = d \times e$	Other	Adjustmen

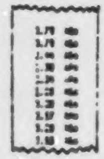
PROBE: Another Way?
NOTES:

#6 4338 ÷ 93	Round	Compatible Numbers	Special Numbers	Other	Adjustment

PROBE: Another way?

NOTES:

PROBLEM	ESTIMATE	STRATEGY			

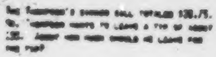
#7 

Rounded to: Compatible Numbers (Grouping) Front-End Adjustment

Dollar Dime

PROBE: How close?

NOTES:

#8 

Rounded and take Compatible Numbers Other Adjustment

15% 10% + $\frac{1}{2}$ 1/7 x 28 1/6 x 30

PROBE: Another way?

NOTES:

#9

Rounded to: Compatible Numbers Front-End Adjustment

Dollar Dime

PROBE: Could you buy them for $37?

Could you buy them for (Response)?

NOTES:

#10

Reasonable? Rounded to: Checked ones place? Adjustment

YES NO 20x40 20x50 30x50

PROBE: How did you know?

NOTES:

Where did you learn to estimate?

What kind of problems are the hardest?

Hints:

Interviewer Comments:

1.

$$\begin{array}{r} 37\ 419 \\ 46\ 765 \\ 41\ 045 \\ +\ 34\ 974 \\ \hline \end{array}$$

PROBE: DO YOU THINK THE ACTUAL ANSWER IS ABOVE OR BELOW YOUR ESTIMATE?

2.

$$\frac{12}{13} + \frac{7}{8}$$

PROBE: DO YOU THINK THE ACTUAL ANSWER IS ABOVE OR BELOW YOUR ESTIMATE?

3.

$$2\frac{3}{5} + 7\frac{5}{6} + \frac{9}{10}$$

PROBE: "IS YOUR ESTIMATE MORE THAN THE EXACT ANSWER?"

4.

$$.943 \times 8076$$

PROBE: DO YOU THINK THE ACTUAL ANSWER IS ABOVE OR BELOW YOUR ESTIMATE?

5.

$$\frac{846 \times 9}{43}$$

PROBE: IS THERE ANOTHER WAY YOU COULD DO IT?

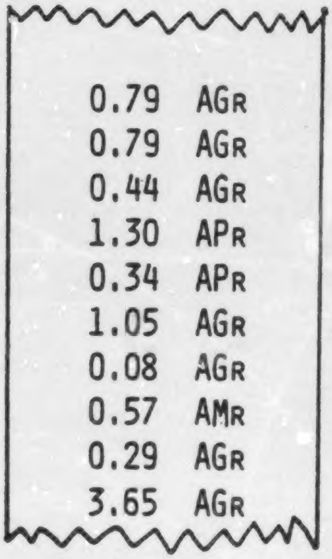
6.

$$4338 \div 93$$

PROBE: IS THERE ANOTHER WAY YOU COULD DO IT?

7.

HERE IS A GROCERY TICKET WHICH HAS NOT YET BEEN TOTALED. ESTIMATE THE TOTAL.



0.79	AGR
0.79	AGR
0.44	AGR
1.30	APR
0.34	APR
1.05	AGR
0.08	AGR
0.57	AMR
0.29	AGR
3.65	AGR

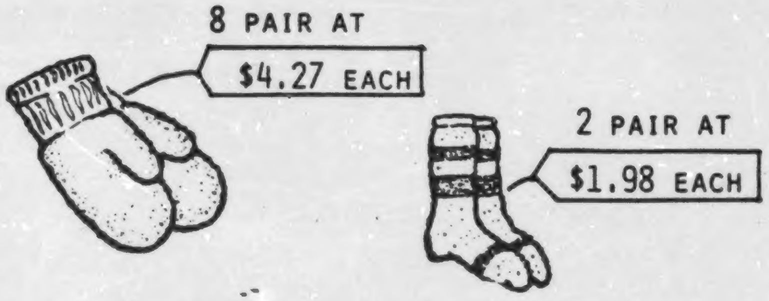
PROBE: HOW CLOSE DO YOU THINK YOUR ESTIMATE IS?

8.

THE THOMPSON'S DINNER BILL TOTALED $28.75. MR. THOMPSON WANTS TO LEAVE A TIP OF ABOUT 15%. ABOUT HOW MUCH SHOULD HE LEAVE FOR THE TIP?

PROBE: IS THERE ANOTHER WAY YOU COULD DO IT?

9.



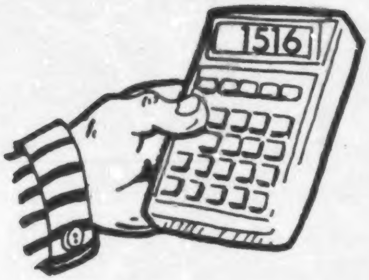
About how much would this cost?

Probe: If RESPONSE < $37, ask Could you buy them for $37?

If RESPONSE > $37, ask Could you buy them for RESPONSE?

10.

22 x 48



Is this calculator result reasonable?

APPENDIX G

Forms Used to Evaluate Instructional Materials

NSF Estimation Project
Lesson Evaluation Form

Teacher: ______________________________

School: ______________________________

Grade: __________ Lesson Number: ____

Date of Lesson: ____________________

1. About how many minutes where spent on this lesson? ______________

2. How did the lesson go? Really Well ○ O.K. ○ Some Problems ○ Poorly ○

   Comments/suggestions for improvement: ______________________________

   ____

   ____

3. Were the teacher notes helpful? Yes ○ No ○ Not Sure ○

   Comments/suggestions: ______________________________________

   ____

4. Were the transparencies effective? Yes ○ No ○ Not Sure ○

   Comments/suggestions for improvement: ______________________________

   ____

5. Please comment on, or make suggestions (as needed).

   a) Overall student reaction/responses: ____________________________

   ____

   ____

   b) Highlight trouble spots or significant strengths: ______________

   ____

   ____

   c) Student exercise sheets: __________________________________

   ____

Teacher: ____________________

Grade: _____ School: ____________

SIXTH GRADE MINI-LESSON/MAINTAINENCE FEEDBACK

Mini-Lesson or Maintainence Sheet	Date	Time Required	Comments
(after Lesson 3)			
ML 1: Compatible # (+)			
ML 2: Compatible # (+)			
MAINTAINENCE #1			
(after Lesson 4)			
MAINTAINENCE #2			
ML 3: M. A. Multiply			
(after Lesson 6)			
ML 4: Nice Numbers			
MAINTAINENCE #3			
ML 5: M.A. Multiply			
MAINTAINENCE #4			
(after Lesson 8)			
MAINTAINENCE #5			

Teacher: ____________________

Grade: __________ School: __________

SIXTH GRADE MINI-LESSON/MAINTENANCE FEEDBACK

Mini-Lesson or Maintenance Sheet	Date	Time Required	Comments
(after vacation)			
MAINTENANCE #6			
(after Lesson 8)			
ML 6: Compatible # (-)			
(after Lesson 10)			
MAINTENANCE #7			
ML 7: Est. $\frac{a \times b}{c}$			
MAINTENANCE # 8			
ML 8: M.A. Add 10s, 100s			
ML 9: M.A. 35+20 35-22			
MAINTENANCE #9			
M.L. 10: M.A. 41-10 42-13			
MAINTENANCE #10			
ML 11: Frac. close to 1,½,0			
ML 12: Est. Sum of Frac.			
ML 13: Est. Sum of Mix. No.			

SIXTH GRADE MINI-LESSON/MAINTENANCE FEEDBACK

(after Lesson 10)

Mini-Lesson or Maintenance Sheet	Date	Time Required	Comments
MAINTENANCE #11			
ML 14: M.A. ¼ of 24			
ML 15: Est. ¼ of 23			
MAINTENANCE #12			
ML 16: Est. +,- Dec.			
ML 17: Dec. close to 1,10			
ML 18: Est. × Dec. (Mice)			
MAINTENANCE #13			
ML #19: M.A. .25 × n			
ML 20: Est. .25 × 79			

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Teacher: ______________________

Grade: ____ School: ____________

SEVENTH GRADE MINI-LESSON/MAINTAINENCE FEEDBACK

(after Lesson 1)

Mini-Lesson or Maintainence Sheet	Date	Time Required	Comments
ML 1: Averaging			
ML 2: F. E. Subtraction			
ML 3: Compatible # (+)			
MAINTAINENCE #1			
ML 4: M. A. Multiply			

(after Lesson 2)

ML 5: M. A. Multiply			
MAINTAINENCE #2			
ML 6: M. A. Multiply			
MAINTAINENCE #3			
ML 7: Nice Nos.			
MAINTAINENCE #4			
ML 8: Count Digits			
MAINTAINENCE #5			

SEVENTH GRADE MINI-LESSON/MAINTAINENCE FEEDBACK

Mini-Lesson or Maintainence Sheet	Date	Time Required	Comments
(after Lesson 3)			
ML 9: Compatible # (-)			
(after Lesson 4)			
MAINTAINENCE #6			
MAINTAINENCE #7			

Teacher: ____________________

Grade: __________ School: __________

SEVENTH GRADE MINI-LESSON/MAINTENANCE FEEDBACK

(after vacation)

Mini-Lesson or Maintenance Sheet	Date	Time Required	Comments
MAINTENANCE #8			
ML 10: Est. $\frac{a+b}{c}$			
ML 11: Frac close to 1, $\frac{1}{2}$, 0			
MAINTENANCE #9			

(after Lesson 5)

ML 12: M.A. +/- Mixed #'s			
ML 13: Est +/- Mixed #'s			
ML 14: More Rounding			

(after Lesson 6)

ML 15: M.A. x Mixed #'s			
MAINTENANCE #10			

(after Lesson 7)

MAINTENANCE #11			

(after Lesson 8)

ML 16: Dec. x 10, 100			
MAINTENANCE #12			
ML 17: Dec. ÷ 10, 100			

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SEVENTH GRADE MINI-LESSON/MAINTENANCE FEEDBACK

Mini-Lesson or Maintenance Sheet	Date	Time Required	Comments
(after Lesson 9)			
ML 18: Quo. >,< 1			
MAINTENANCE #13			
ML 19: Frac. ⟶ Percents			
ML 20: M.A. 1%,10%			
(after Lesson 10)			
MAINTENANCE #14			
[redacted]			
MAINTENANCE #15			

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Teacher: ____________________

Grade: ______ School: ______________

MINI-LESSON/MAINTENANCE FEEDBACK

Mini-Lesson or Maintenance Sheet	Date	Time Required	Comments
#1 - Adding to 100			
#2 - Adding Rounded Numbers			
#3 - Adding Nines			
#4 - Chains of of Addition and Subtraction			
#5 - Compatible Numbers - Addition			
#6 - Multiples of Ten			
MAINTENANCE SHEET 1			
#7 - Compatible Numbers - Multiplication			
#8 - Number of Digits in Products			
#9 - Multiplication Short Cut			
MAINTENANCE SHEET 2			
#10 - Compatible Numbers - Multiplying by 25 & 50			
#11 - Number of Digits in Quotient			
MAINTENANCE SHEET 3			

Grade 8

Teacher: ____________________

Grade: ____ School: ____________

MINI-LESSON/MAINTENANCE FEEDBACK

Mini-Lesson or Maintenance Sheet	Date	Time Required	Comments
#12 - Predicting Number of Digits			
#13 - Powers of Ten (x, ÷)			
MAINTENANCE SHEET 4			
#14 - Fractions Near 0, ½, & 1			
#15 - Fractions - More or Less			
#16 - Fractions Near ½ and 1			
#17 - Fraction - Decimal Equivalents			
#18 - Exploring Nice Fractions			
#19 - Exploring Fractions			
MAINTENANCE SHEET 5			
#20 - Fraction - Decimal Approximations			
#21 - Placing the Decimal Point			
MAINTENANCE SHEET 6			
#22 - Percent - Fraction Equivalents			
#23 - Special Percents			
MAINTENANCE SHEET 7			

APPENDIX H

Summary of Presentations Made to Disseminate Project Materials

DISSEMINATION EFFORTS

Many efforts have been made to alert the mathematics education community to the availability of instructional materials from this project. In addition to announcements and articles related to the project, a number of presentations were made at professional meetings. Here is a summary of presentations at state, regional and national meetings by project staff.

Northeast Missouri District Teachers Meeting
Kirksville, Missouri October 1982

National Council of Teachers of Mathematics
Topeka, Kansas October 1982

Northwest Council of Teachers of Mathematics
Portland, Oregon October 1982

Kansas City District Teachers Meeting
Kansas City, Missouri November 1982

New Trier Township Teachers Meeting
Winnetka, Illinois March 1983

National Council of Supervisors of Mathematics
Detroit, Michigan April 1983

AERA-Special Interest Group for Research in Mathematics Education
Detroit, Michigan April 1983

National Council of Teachers of Mathematics
Detroit, Michigan April 1983

Mathematics Club of Greater St. Louis
St. Louis, Missouri September 1983

Parkway School District Teachers Meeting
Chesterfield, Missouri September 1983

Texas Council of Teachers of Mathematics
Austin, Texas October 1983

Florida Council of Teachers of Mathemaics
Jacksonville, Florida October 1983

National Council of Teachers of Mathematics
Omaha, Nebraska October 1983

Kentucky Mathematics Teachers Meeting
Louisville, Kentucky October 1983

Illinois Council of Teachers of Mathematics
Normal, Illinois November 1983

National Council of Teachers of Mathematics
Philadelphia, Pennsylvania November 1983

Southern California Council of Teachers of Mathematics
Long Beach, California November 1983

Virginia Supervisors of Teachers of Mathematics
Richmond, Virginia March 1984

Illinois Council of Teachers of Mathematics
Charleston, Illinois March 1984

National Council of Teachers of Mathematics
Dekalb, Illinois March 1984

Virginia Council of Teachers of Mathematics
Richmond, Virginia March 1984

National Council of Supervisors of Mathematics
San Francisco, California April 1984

National Council of Teachers of Mathematics
San Francisco, California April 1984